Assignment 1 Solutions

1 Problem 1

1.1 Part a

In order to calculate the rate of sedimentation during each of these time periods, we need to know how much material was deposited and the time over which it was deposited. In terms of units, we don't have the area over which this material was laid down, so we can't calculate a volume rate of deposition, but we do have vertical accumulation in feet, so let's use that.

In each case, we'll divide the thickness of sediment by the time over which it was deposited. For consistency, let us assume that the age listed for each layer corresponds to the *upper* boundary of that layer. In cases where the geologic boundary does not match up with an age, we can estimate the boundary's age based on its relative placement within known bounds. For example, the Permian boundary falls somewhere within the 1000' layer of rock dated at 285-315 My: it looks like it's about 25% into that layer, so we can use an age that's 25%, or 7.5 My, into that layer (292.5), and a corresponding thickness of 250' (25% of 1000').

Permian:

Thickness = 350' (thickness of layer 1) + 250' (thickness of layer 2) + 300' (thickness of layer 3) + 300' (thickness of layer 4) + 250' (portion of layer 5 that falls within the Permian) = 1450 feet

Age range = 292.5 (Age of the first rock layer deposited during the Permian... see explanation above) – 270 (Age of the most recently deposited rock layer in the Permian) = 22.5 million years

Sedimentation rate = 1450 feet / 22.5 My = **65 feet / My**

Pennsylvanian:

Thickness = 750' (the remaining portion of layer 5 that is not part of the Permian)

Age range = 315 (age of first rock deposited in the Pennsylvanian) – 292.5 (our estimated Pennsylvanian-Permian boundary) = 22.5 My

Sedimentation rate = 750 feet / 22.5 My = **33.3 feet / My**

Mississippian:

This one's a bit tricky because we have a range of rock thicknesses in layer 6, but we can estimate a lower bound on the sedimentation rate by using the minimal thickness (0 feet), and an upper bound using the maximal thickness (75 feet). In both cases, we can use 320 My as the most recent age.

Thickness (lower bound) = 0' (lower bound thickness of layer 6) + 500' (thickness of layer 7) = 500 feet

Thickness (upper bound) = 75' (upper bound thickness of layer 6) + 500' (thickness of layer 7) = 575 feet

Age range = 385 (age of last rock layer deposited before the Mississippian) – 320 (age of last rock layer deposited in Mississippian) = 65 My

Sedimentation rate (lower bound) = 500' / 65 = **8 feet / My**
Sedimentation rate (upper bound) = 575' / 65 = **9 feet / My**

Cambrian:

As with the Mississippian, we have a range of thicknesses in layer 11 to deal with, and we'll handle it the same way by providing an upper and lower bound on sedimentation rate. We also have variation that comes from the layer age value of the last layer of the proterozoan, with layer 12 showing an age of less than 740 My. This means that at the very earliest, layer 12 was deposited 740 My, but the youngest possible interpretation is 525 My (the age of the layer on top; we know layer 12 had to be in place by 525 My). We can get a range of sedimentation rates using these bounds on age range.

Thickness (lower bound) = 450' (thickness of layer 9) + 350' (thickness of layer 10) + 0' (lower bound thickness of layer 11) = 800 feet

Thickness (upper bound) = 450' (thickness of layer 9) + 350' (thickness of layer 10) + 200' (upper bound thickness of layer 11) = 1000 feet

Age range (lower bound) = 525 (age of first layer of the Cambrian) – 505 (age of last layer deposited in Cambrian) = 20 My

Age range (upper bound) = 740 (age of oldest possible interpretation for last rock deposited in Proterozoic) – 505 (age of last layer deposited in Cambrian) = 235 My

Sedimentation rate (lower bound) = 800' / 235 = **3.4 feet / My**
Sedimentation rate (upper bound) = 1000' / 20 = **50 feet / My**

This is quite a large range in sedimentation rates, which goes to show how difficult it can be to make precise measurements in complex systems!
Proterozoic:

Again, our variation comes from the layer age value, with layer 12 showing an age of less than 740 My. This means that at the very earliest, layer 12 was deposited 740 My, but the youngest possible interpretation is 525 My (the age of the layer on top; we know layer 12 had to be in place by 525 My). We can get a range of sedimentation rates using these bounds on age range.

Thickness = 200' (thickness of layer 12) + 5200' (thickness of layer 13) + 370' (thickness of layer 14) + 6800' (thickness of layer 15) = 12570'

Age Range = 1200 (age of first layer deposited in Grand Canyon Supergroup) – 740 (age of oldest possible interpretation for last rock deposited in Proterozoic) = 460 My

Age Range = 1200 (age of first layer deposited in Grand Canyon Supergroup) – 525 (age of youngest possible interpretation for last rock deposited in Proterozoic) = 675 My

Sedimentation rate (lower bound) = 12570 / 675 = **18.6 feet / My** Sedimentation rate (upper bound) = 12570 / 460 = **27.3 feet / My**

Our variation is from about 8 feet per My to 65 feet / My. Several factors could have caused this variation, including the following:
Compaction: Over time, as more sediment is deposited on top of earlier layers, those layers will be compacted, squeezing them into a smaller vertical range. Thus, deeper layers that experience more overburden may give exhibit artificially small sedimentation rates.

Lithology: The type of material being deposited during different time periods could account for some of the variation. For example, rock that is easier to erode could have been deposited at a higher rate than rock that is harder to erode, even under the same environmental conditions.

Climate: During a period of increased rain, we would expect higher deposition rates than during drier periods, as there would be more runoff from surrounding areas.

Figure 1

Tectonics: Erosion also increases with the vertical profile of surrounding terrain – steeper mountains erode quicker, leading to faster deposition rates. A period of more rapid tectonic uplift surrounding (but not including) the area we're looking at would generate larger deposition rates.

Regional flora and fauna: Plants, and to a lesser extent animals, can influence erosion rates. For example, a hillside with lots of large, well-rooted trees will retain its soil better than one without, leading to lower sedimentation rates.

Others? Probably! Just explain how different environmental or geologic conditions may alter our calculated sedimentation rates.

1.2 Part b

The goal of this problem is simply to make sure that when we stare at the map and remember that plates are in motion, we don't forget to ask ourselves whether all the numbers when put together actually make sense. In this case, I look at a map with a 20 km scale bar and estimate that Espanola has moved 200 km in 3.5×10^6 years (see Figure 1. Using the most naive estimate this means the speed is

$$\text{speed} \approx \frac{2 \times 10^7 \text{ cm}}{3 \times 10^6 \text{ years}} \approx 7 \text{ cm/year}. \tag{1.1}$$

We now bring this thinking to bear on the Hawaiian Islands. Next time you fly into Hawaii, look out the window. No matter which island you are flying to, if the day is clear you will see the line of volcanic islands out your window and it is an impressive sight.

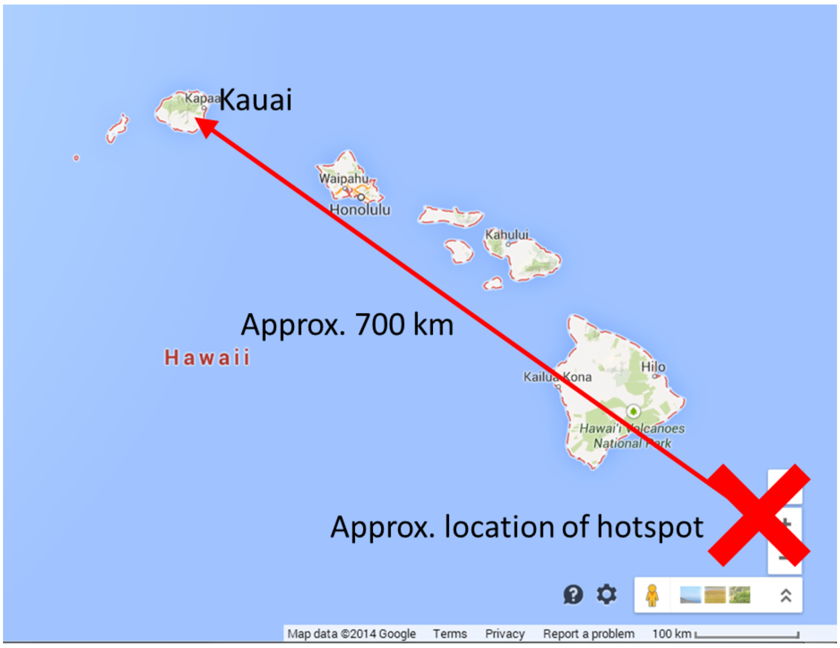

Figure 2

We will say that Kauai is 5×10^7 cm away from the hot spot, estimated from Figure 2. Given that it is traveling the same speed as Espanola, this means its age is given by

$$\text{age of Kauai} \approx \frac{7 \times 10^7 \text{ cm}}{7 \text{ cm/year}} \approx 10^7 \text{ years} \tag{1.2}$$

As with every estimate on this homework, there are many simplifying assumptions that could be improved on and there are many "facts" that could be refined. However, from the standpoint of the "feeling for the numbers" mindset, this estimate gives us the main picture: there is a cm/year speed in play for plates and this means that macroscopic geographical motions take place on the millions of years time scale.

2 Problem 2

2.1 Part a

The case of K-Ar dating is much easier conceptually because we don't have to deal with an unknown amount of initial Ar since we assume that upon solidification of the rock, all the Ar gas is liberated. The dynamics of the K is simple and can be written as

$$\frac{dN_K}{dt} = -k_K N_K. \tag{2.1}$$

Just to remind everyone what such an equation says, it is telling us that at every instant, every nucleus of ^{40}K has exactly the same probability of decaying in the next increment of time Δt. We can solve for the amount of K as a funciton of time as

$$N_K(t) = N(0)e^{-k_K t}. \tag{2.2}$$

By conservation of mass, we know that every K that decays becomes an Ar, and hence the equation of motion for the number of Ar atoms is

$$\frac{dN_{Ar}}{dt} = k_N N_K. \tag{2.3}$$

Because every A is produced by the decay of a K, we can use the solution for $N_K(t)$ to write an equation for $N_A(t)$, giving us

$$N_{Ar}(t) = N_K(0)(1 - e^{-kt}). \tag{2.4}$$

We can now take our two solutions and divide them with the result that

$$\frac{N_{Ar}(t)}{N_K(t)} = \frac{N_K(0)(1 - e^{-k_K t})}{N_K(0)e^{-k_K t}} = \frac{(1 - e^{-k_K t})}{e^{-k_K t}} \tag{2.5}$$

All that is left now is to solve for the unknown age of the rock t. Note that as a result of taking the ratio of the two equations, the unknown initial number of potassium nuclei has dropped out of the problem and we are left with

$$t = \frac{1}{k_K} ln(1 + \frac{N_{Ar}}{N_K}). \tag{2.6}$$

2.2 Part b

In this case, we are interested in the decay of ^{87}Rb into ^{87}Sr. Like in the previous part of the problem, the differential equation describing this process can be written by inspection as

$$\frac{dN_{Rb87}}{dt} = -kN_{Rb87} \tag{2.7}$$

This equation then implies that

$$N_{Rb87}(t) = N_{Rb87}(0)e^{-kt} \tag{2.8}$$

Using conservation of mass, we know that the amount of ^{87}Sr increases with the same exponential form as the ^{87}Rb decays. More precisely, we have

$$N_{Sr87}(t) = N_{Sr87}(0) + (N_{Rb87}(0) - N_{Rb87}(t)). \tag{2.9}$$

Because $N_{Rb87}(0)$ can be rewritten as $N_{Rb87}(t)e^{kt}$, we can rewrite this as

$$N_{Sr87}(t) = N_{Sr87}(0) + N_{Rb87}(t)(e^{kt} - 1). \tag{2.10}$$

Dividing both our solutions by $N_{Sr86}(0)$, we have

$$\frac{N_{Rb87}(t)}{N_{Sr86}(0)} = \frac{N_{Rb87}(0)}{N_{Sr86}(0)} e^{-kt} \tag{2.11}$$

and

$$\frac{N_{Sr87}(t)}{N_{Sr86}(0)} = \frac{N_{Sr87}(0)}{N_{Sr86}(0)} + \frac{N_{Rb87}(t)}{N_{Sr86}(0)}(e^{kt} - 1). \tag{2.12}$$

Note that the quantities we know as a result of our measurements are: $N_{Sr87}(t)$, $N_{Rb87}(t)$ and $N_{Sr86}(0)$. On the other hand, we have no direct access to the quantity $N_{Sr87}(0)$, though we know that this quantity is constant for all minerals that are found in our rock of interest.

We determine k from the half-life of ^{87}Rb (4.88×10^10 years) using the equation

$$N_{Rb87}(t) = N_{Rb87}(0)e^{-kt}, \tag{2.13}$$

where t is the half life. This gives us

$$\frac{N_{Rb87}(0)}{2} = N_{Rb87}(0)e^{-k\times 4.88\times 10^10}. \tag{2.14}$$

This simplifies to

$$\frac{1}{2} = e^{-k\times 4.88\times 10^10} \tag{2.15}$$

Which we solve to give us

$$k = \frac{\ln 2}{4.88 \times 10^10} = 1.42 \times 10^{-11}. \tag{2.16}$$

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In many cases, it is appropriate to take advantage of the smallness of the quantity kt « 1, which permits us to expand the exponential in a Taylor series. Taking the first two terms of the Taylor expansion for an exponential gives us

$$e^{kt} = 1 + kt, \tag{2.17}$$

Allowing us to rewrite the expression as

$$\frac{N_{Rb87}(t)}{N_{Sr86}(0)} = \frac{N_{Sr87}(t)}{N_{Sr86}(0)} + \frac{N_{Rb87}(t)}{N_{Sr86}(0)}(kt). \tag{2.18}$$

Note that this equation follows the form of a linear equation, y = mx + b, where

$$y = \frac{N_{Sr87}(t)}{N_{Sr86}(0)}, \tag{2.19}$$

$$\frac{N_{Rb87}(t)}{N_{Sr86}(0)}, \tag{2.20}$$

$$m = kt, \tag{2.21}$$

and

$$b = \frac{N_{Sr87}(0)}{N_{Sr86}(0)}. \tag{2.22}$$

Note that since our value of interest is t, and this can be obtained from the slope of x plotted against y, we do not need to know the y-intercept and thus do not need to know the starting value for N_{Sr87}.

From Figure 3 in the homework, we can choose two points and estimate the slope to be approximately

$$\frac{0.78 - 0.74}{3 - 1.5} = 0.027. \tag{2.23}$$

Finally, we use our calculation of k and the slope to determine t:

$$t = \frac{0.027}{1.42 \times 10^{-11}} \approx 1900 \text{ million years}. \tag{2.24}$$

In spite of our rough estimates and simplifying assumptions, this agrees quite well with the actual value of 1725 million years!