“Leave it as it is. You cannot improve on it. The ages have been at work on it, and man can only mar it.” - Theodore Roosevelt speaking of the Grand Canyon

1. A feeling for the numbers in evolution

The processes of evolution take place at many different scales in both space and time. The goal of this first problem is nothing more than to “play” with some of the characteristic scales associated with processes in evolution. These estimates are intended to be done using simple arithmetic of the “one-few-ten” variety (i.e. few times few is ten) and to give an order-of-magnitude picture of the phenomenon of interest. Take pride in your results and state and justify (with citations) the assumptions you make carefully and give a simple, intuitive description of how you came to your results. Please don’t report rough estimates with long lists of “significant” figures.

a) One of the early methods that was tried out for dating the age of the Earth was to estimate the rate of sedimentation and to figure out how long certain geological features would have taken to form. (This strategy didn’t end up working for making reliable estimates - see G. Brent Dalrymple, “Ancient Earth, Ancient Skies: The Age of Earth and its Cosmic Surroundings” for an excellent history of the attempt to date the Earth). Using the attached figure of the Grand Canyon (Figure 1), estimate the rate of sedimentation during each of the periods / subperiods listed in the time scale (Permian, Pennsylvanian, Mississippian, Cambrian, and Proterozoic to the base of the Grand Canyon Supergroup). What might have caused the variation you calculate?

(b) One of the reasons that evolution is hard to think about is because of the vast times involved in evolutionary processes. One of the more interesting things we have to remember is the interplay between the dynamics of the earth and the dynamics of the living organisms on earth. For example, we will later learn about the consequences of the closing off of the Isthmus of Panama for evolution. Similarly, the collision of India with the Asian conti-
Figure 1: The Grand Canyon over time.
nent brought an end to the Tethys Sea. In this part of the problem, we will think about such geological processes as they bear on some of our favorite topics from the course.

We begin by thinking about how fast the Galapagos Islands are moving. Note that these islands, like the Hawaiian Islands, are being produced by a “hotspot” that is near the current islands of Isabella and Fernandina. The islands then move in a southeasterly direction towards the coast of South America. Given that the island of Espanola is roughly \(3.5 \times 10^6\) years old, make an estimate for the mean rate at which these islands are moving to the southeast per year. Give your answer in cm/year. Assuming that the speed you found for the Galapagos is typical for island chains, make an estimate of the age of the island Kauai using the same kind of logic. What factors might complicate this comparison, and how would they change this estimate (qualitatively)?

(c) To Think About: some of the animals on the Galapagos appear, from DNA evidence, to have diverged more than \(3.5 \times 10^6\) years ago. How can we reconcile that with the age of the islands?

2. Dating for Geologists and Biologists

One of the most interesting topics in science is how we have learned to probe deep time. In this course, the subject of deep time will rear its head repeatedly and we will spend a lot of time examining how DNA sequence has permitted us to explore deep time in the biological setting. Of course, biology and the dynamics of the Earth are not independent phenomena and the point of this problem is to better understand the details of how scientists have come to terms with figuring out how old the Earth is as well as how old various fossil-bearing strata are. To that end, we will consider a simple model of the radioactive decay process, first for potassium-argon dating methods and then for rubidium-strontium dating methods.

(a) Potassium-argon dating is based upon the decay of \(^{40}\text{K}\) into \(^{40}\text{Ar}\). To a first approximation, this method can be thought of as a simple stopwatch in which at \(t = 0\) (i.e. when the rocks crystallize), the amount of \(^{40}\text{Ar}\) is zero, since in gaseous form it is presumed that all of the argon has escaped. Given that the decay rate of \(^{40}\text{K}\) is \(k_K = 5.8 \times 10^{-11} \text{ yr}^{-1}\), implying a half-life of
1.25 \times 10^9 \text{ years}, write a differential equation for the amount of the relevant isotopes of both K and Ar as a function of time. Solve these differential equations for \( N_K(t) \) and \( N_{Ar}(t) \) and find an expression for the lifetime of the rock of the form \( t = t(k_K, N_K(t), N_{Ar}(t)) \). Explain how one would use this method and make sure that your expression only involves quantities that one can measure now (i.e. your expression cannot depend upon how much of some particular nucleus was present at the time of crystallization).

(b) A second deep-time stopwatch that is used often is the decay of \(^{87}\text{Rb}\) into \(^{87}\text{Sr}\) with a half-life of \(1.251 \times 10^9\) years. However, unlike in the case of Ar described in the previous part of the problem, we cannot ignore the initial concentration of the rubidium and strontium at the time that the rock crystallizes. To get around this problem of our ignorance of initial conditions, a very clever method using “isochrons” was developed and in this part of the problem, we will derive the properties of such isochrons and then use them to produce a simple estimate of the age of a rock. See Figure 2 to remind yourself of the meaning of the isochrons. First, for notation. We will consider three functions, \( N_{^{87}\text{Rb}}(t) \), \( N_{^{87}\text{Sr}}(t) \) and \( N_{^{86}\text{Sr}}(t) \), telling us the number of \(^{87}\text{Rb}\), \(^{87}\text{Sr}\) and \(^{86}\text{Sr}\) present at time \( t \). Note that the isotope \(^{86}\text{Sr}\) is stable and hence that \( N_{^{86}\text{Sr}}(t) = N_{^{86}\text{Sr}}(0) \). That is, the amount of \(^{86}\text{Sr}\) that a rock starts with at the time of crystallization is the amount that will be present from then on (assuming there are no losses by other mechanisms). Note also that what we actually measure is these three functions at the time \( t \) after the rock has crystallized. What we need is an expression for the elapsed time that depends upon these measurable quantities rather than quantities which we have no access to such as the initial concentrations.

Begin by noting that we have a constraint implied by the conservation of mass of the form

\[ N_{^{87}\text{Rb}}(t) + N_{^{87}\text{Sr}}(t) + N_{^{86}\text{Sr}}(t) = N_{^{87}\text{Rb}}(0) + N_{^{87}\text{Sr}}(0) + N_{^{86}\text{Sr}}(0). \]

(1)

Make sure you explain simply why this constraint is in effect. Further, we know that \( N_{^{86}\text{Sr}}(t) = N_{^{86}\text{Sr}}(0) \). Write differential equations for \( N_{^{87}\text{Rb}}(t) \) and \( N_{^{87}\text{Sr}}(t) \) using only the decay constant \( k_{Rb} \) which characterizes the rate of decay of \(^{87}\text{Rb}\). Solve both of these equations and thereby obtain expressions for \( N_{^{87}\text{Rb}}(t) \) and \( N_{^{87}\text{Sr}}(t) \), noting that the solutions depend upon \( N_{^{87}\text{Rb}}(0) \) and \( N_{^{87}\text{Sr}}(0) \). Normalize your solution for \( N_{^{87}\text{Sr}}(t) \) by dividing by \( N_{^{86}\text{Sr}}(0) \).
Radiometric dating indicates that the earth is 4.6 billion years old.

Figure 2: The concept of isochrons. (A) A rock containing a number of different minerals, each of which has its own distinct ratio of $^{87}$Rb to $^{86}$Sr. (B) The nuclear reaction that transforms Rb into Sr. (C) Schematic of how the amount of $^{87}$Sr/$^{86}$Sr changes over time. From Zimmer and Emlen, “Evolution”, chap. 3.

In light of these results, show that

$$\frac{N_{Sr87}(t)}{N_{Sr86}(0)} = \frac{N_{Sr87}(0)}{N_{Sr86}(0)} + \frac{N_{Rb87}(t)}{N_{Sr86}(0)}(e^{kt} - 1).$$

(2)

Given the value for $k$ that you can deduce from the half-life of the decay reaction of Rb into Sr, is it justified to make a Taylor series approximation for $e^{kt}$ and if so, how does that simplify the expression given above.

Interpret this solution specifically by appealing to Figure 2 (also, note that we use the rate constant $k_{Rb}$ whereas the figure uses $\lambda$). What does this mean about the data that we will find for all of the minerals within a rock that crystallized some time $t$ ago? Using the data provided in Figure 3, pick two data points and use them to determine the age of this Precambrian granite body. This is a scratch and sniff estimate - just pick two points that seem most convenient and make approximate values of where they sit in the ($^{87}$Rb/$^{86}$Sr, $^{87}$Sr/$^{86}$Sr) plane. Explain how you made the unknown initial value of $^{87}$Sr/$^{86}$Sr drop out of the problem and provide an estimate for the age of the rock.
Figure 3: Data from Rb-Sr dating.