

Physical biology of the cell (2015)

Practice problems from Sanjoy

Several problems ask for your answer in the form 10^x . This form is a reminder that great accuracy is neither needed nor wanted. An answer to the nearest half-power of 10 is good enough (e.g., $10^{2.5}$ or 10^{-1}).

Warmups

1 Air mass

Estimate the mass of air in the classroom.

$$10^{\boxed{}} \text{ kg}$$

2 Density of uranium

Using our rule of thumb for densities, estimate the density of uranium (atomic number 92, atomic mass 238), and then compare to the actual density.

$$\boxed{} \text{ g cm}^{-3}$$

What do you conclude about the typical spacing between uranium atoms?

3 Direct practice with half-integer powers of 10

Here is a randomly generated multiplication estimation problem to practice avoiding calculators:

$$985 \times 385 \times 721 \times 319. \tag{1}$$

Estimate the product by rounding each factor to the nearest half-power of 10, e.g. $10^{3.5}$ (also known as a few $\times 10^3$) or $10^{2.0}$ and multiplying them in your head. Then compare with the exact value (for which you'll need the calculator).

$$10^{\boxed{}}$$

4 Fractional change

a. What is the fractional change going from 80 to 84?

b. What is the approximate fractional change going from 80^2 to 84^2 ?

c. Therefore estimate 84^2 (without a calculator).

d. Check this estimate against the exact result.

Problems

5 Gas stations

Estimate the number of gas stations in the United States.

10

6 Cooking time

Roughly speaking, an object has cooked once the hot surface temperature has diffused into the object. An egg ($l \approx 2$ inches) cooks in roughly 10 minutes. How long should it take to cook a turkey ($l \approx 10$ inches)? Check your answer by asking anyone who has cooked a turkey for Thanksgiving.

hours

7 High winds

At roughly what wind speed is the force on your body from the wind approximately equal to your weight?

m s^{-1}

8 Random walk in two dimensions

In class, we studied a random walk in one dimension (on a line) and found that $\langle x^2 \rangle$ increased by 1 with each time step—the crucial property in our derivation of the diffusion time and then Fick's law. As our example, we took a particle at $x = 5$, so $\langle x^2 \rangle = 25$. After one step, it is equally likely to be at $x = 4$ or $x = 6$, so

$$\langle x^2 \rangle = \frac{1}{2} \times 4^2 + \frac{1}{2} \times 6^2 = 26, \quad (2)$$

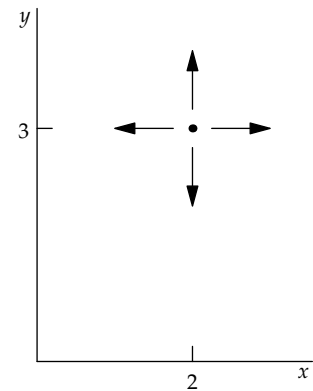
which is $25 + 1$.

In this problem, you test whether this property holds in two dimensions. Start a random-walking particle at $\mathbf{r} = (2, 3)$.

- a. What is $\langle r^2 \rangle$?

- b. On each time step, the particle walks one unit up, down, left, or right (with equal probability). After one time step, what is $\langle r^2 \rangle$?

- c. (Optional!) If, instead, the particle walks one unit in one of the four *diagonal* directions, what is $\langle r^2 \rangle$?



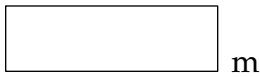
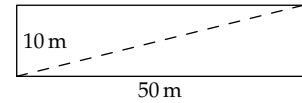
9 Comfortable outside temperature

In class, we estimated a 600-watt heat loss outside on a winter day (0°C), assuming a skin temperature of 30°C and wearing only a very long 2-millimeter-thick T shirt. What outside air temperature would lead to a comfortable 60-watt heat loss? Is that estimate reasonably consistent with experience?

 $^\circ\text{C}$

10 Diagonal path

A large rectangular garden is 50 meters by 10 meters. Find, to the nearest meter, the length of the diagonal path across the garden (without using a calculator).

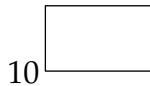


Before making the calculation, guess whether it will be closest to 50, 55, or 60 meters.

11 Population growth

Assume that the world population, now 7 billion, has been growing at 1% per year for the last 700 years.

- a. What was the world population in 1315? Don't use a calculator!



- b. The actual world population in 1300 was around 400 million (according to the US Census Bureau). Does that information mean we under- or overestimated the growth rate (when we used 1%)? Optional: What is a better estimate of the growth rate?