The Black Box, or the Significance of $r = \left(\frac{E}{\rho} t^2 \right)^{\frac{1}{5}}$

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To the reader. For many years, a certain anecdote has circulated through university physics departments, passed down from one generation of graduate students to the next. About 50 years ago, a man named Geoffrey I. Taylor estimated the explosive power of the first atomic bomb, the one tested in the desert of New Mexico in July 1945. When Taylor made his estimate, the bomb's power was a military secret. Time and declassification have since revealed Taylor's estimate to be highly accurate. His method of dimensional analysis is why the story persists. He used what was available to him: intuition and photographs of the explosion appearing in a popular magazine. With that, and that alone, he uncovered one of America's deepest secrets [1]. This story recounts Taylor's approach through the fictional character of Milvia Capelli and the equally fictional events composing her life.

The aim of this piece is to profile the physicist at work, struggling with a problem that initially appears intractable. Just as Sherlock Holmes uses deduction to break open mysteries insoluble to others, Milvia -- like all physicists -- uses mathematical reasoning. Hopefully, this story will illustrate the nature of that approach. Elementary algebra and memory of freshman physics are needed to follow the series of equations she uses to discover the answer. The laboratory notebook is essentially the professional diary of the working scientist. Consequently, six full pages of this story appear as laboratory notebook pages, each one labeled "University of Chicago Laboratories". Most of the equations in this story appear in these pages. They allow you, the reader, to follow her thinking as it happened. These notebook pages are integrated into the story. For the reader interested in better understanding Milvia's (that is, G.I. Taylor's) approach, I have included an appendix. This five page appendix is *not* part of the story, and is not written as a short story. Read it to see Milvia's approach applied to an analogous, but simpler, problem. In physics, a good approach finds many different, sometimes unexpected, applications.

The larger story into which this profile fits. This profile is the central part of a larger story in which the full plot emerges. In brief, it is 1955. The Cold War burns and international tensions are running high. Analysis of radioactive fallout suggest to some US scientists that the Soviets are way ahead in developing a cobalt-60 super bomb. They are testing it. President Dwight D. Eisenhower must act. Lead by Secretary of State John Foster Dulles, hawks in his cabinet urge the President to take pre-emptive action. Attack the Soviet Union now, they urge him, using all

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[1] A tersely told version of the anecdote appears at the beginning of chapter six of David Goodstein's book *States of Matter* (Dover, 1985), a textbook on statistical thermodynamics. Another account is the article "Modern Classical Physics Through the Work of G. I. Taylor" by Michael P. Brenner and Howard A. Stone. It appears in *Physics Today on the Web*, May 2000, http://www.aip.org/pt/may00/taylor.htm.

the nuclear bombs we possess. A wise, old soldier, Eisenhower knows better. Secretly, he devises a way of testing his cabinet, and the quality of their advice. It is a ploy that becomes apparent at the end of the larger story.

A second crisis then emerges. Photographs of a nuclear bomb explosion appear in *Life* magazine. Illustrated is that first bomb test in New Mexico, or is it? Eisenhower's Secretary of Defense Charles Wilson breaks the bad news: pictures and the negatives of the New Mexico bomb test were destroyed years earlier by order of the Truman Administration. And *Life* magazine cannot trace the source of the pictures. And the Pentagon's Office of Remote Photographic Surveillance and Assessment concludes that these pictures could have been taken either in the New Mexico desert, or in the Chinese Takla Makan desert. *What then is pictured?* Could it be the Soviet super bomb, the pictures intentionally leaked by the Soviet Union to cow the American military? Some of Eisenhower's cabinet members think so. They argue that uncertainty about the pictures's origin provides further justification for attacking the Soviet Union. To make things worse, details about this embarrassing conundrum are leaked to a reporter for the *Chicago Tribune*. A story appears on the front page of the Sunday, 4 December 1955 edition.

Eisenhower demands a second opinion and turns to American physicist J. Robert Oppenheimer, or "Oppy" as his friends call him. A controversial figure, Oppenheimer is a famed nuclear physicist who lead the successful American effort to develop the atomic bomb during World War II. Yet a decade later, at the time of this story, Oppenheimer is without a security clearance. Questions about his past ties to persons linked to the Communist Party lead Eisenhower Administration in 1953, and the Atomic Energy Commission in 1954, to revoke Oppenheimer's security clearance. Seeking a truly different perspective, and perhaps feeling contrite, Eisenhower approaches Oppenheimer, know to his friends as "Oppy", for an independent assessment of the photographs and their origin. Oppy subsequently delegates the problem to a junior colleague of his, 28 year old Milvia Capelli, Assistant Professor of Physics at the University of Chicago. Oppy calls her in advance. "Milvia, can you come up with an estimate of the bomb yield in no more than 48 hours?" One could tell much about the photographs, Oppy reasons, with a good estimate the explosive force illustrated. "This is urgent. I have posted a package of documents that should help." As Milvia soon discovers, those documents reflect only what is publicly available. That is all Oppy has access to.

In the profile below, Milvia takes the same approach taken by G.I. Taylor and, like Taylor, produces a surprisingly accurate estimate of the bomb yield. Milvia works from the magazine photographs. She tapes them into page 79 of her laboratory notebook [2]. Like any Assistant Professor, Milvia needs to prove herself to her colleagues. The need is all the more acute for her, the first woman to make a tenure-track position in her department. Her research is based on computer simulation. She needs computer time, a very scarce and precious commodity in 1955. Yet it remains out of her reach. She is not a big name scientist, not a member of the old boys networks common in academia. At the end of the larger story, Eisenhower, his cabinet, and Oppy too are amazed to learn just how accurate her estimate of the bomb yield is. It resolves the controversy about the origin of the pictures. It confirms Eisenhower's resolve not to attack the Soviet Union. It also makes Milvia's name and wins her some needed computer time too.

[2] Credit: Los Alamos National Laboratory. The pictures were declassified long ago.

Estimation of Explosive Yield

Tuesday, 9:30 am, 6 December 1955.
A digression from the usual. Yesterday evening JRO called from Berkeley with an urgent matter though he said little in detail about it! Something about estimating bomb blast energy. He needs an answer of some kind by tomorrow afternoon. "48 hours", he said. He posted "critical documents" for me to the main post office downtown. This morning, against Chicago's bitter cold (-21 C wind chill), I schlepped up there to get it. Those documents turned out to be misc. magazine and newspaper clippings! Or, mimeos with "declassified" stamped on them. Is this a joke?! Is this a take-home examination? I thought I was done with graduate school.

Tuesday, 6:12 pm.
Spent a fruitless day trying to make headway with a thermodynamic / spectroscopic approach. From nuclear binding energy and critical mass, estimate the total energy released. Or at least a lower bound. Assume that most or all of it will be electromagnetic radiation. Roughly estimate the spectral distribution of radiation using Planck's black body theory. Radiation in the microwave and infrared region will interact with matter most strongly and be responsible for destructive mechanical effects (i.e., propagation of shock wave) of the explosion. That's what JRO wants, I think. Other radiation (X-rays, gamma rays) interact much less and will escape into space. (Or will they? What about radiative transport where X-ray photons fall down toward the microwave spectrum through cycles of absorption and reemission?) So how much energy, then, is partitioned into the microwave and IR regions -- directly, and secondarily through RT? The explosion presumably involves either nuclear fission or fusion.

Fission bomb - what I need to know: U-235 or Pu-239?, critical mass (or density?), implosion mechanism (spherical implosion is easiest to model but most difficult, I suspect, to achieve in practice). Also, some means of getting at the distribution of neutron temperature. Most of these parameters must be classified. In any case, the package of clippings is no help at all!

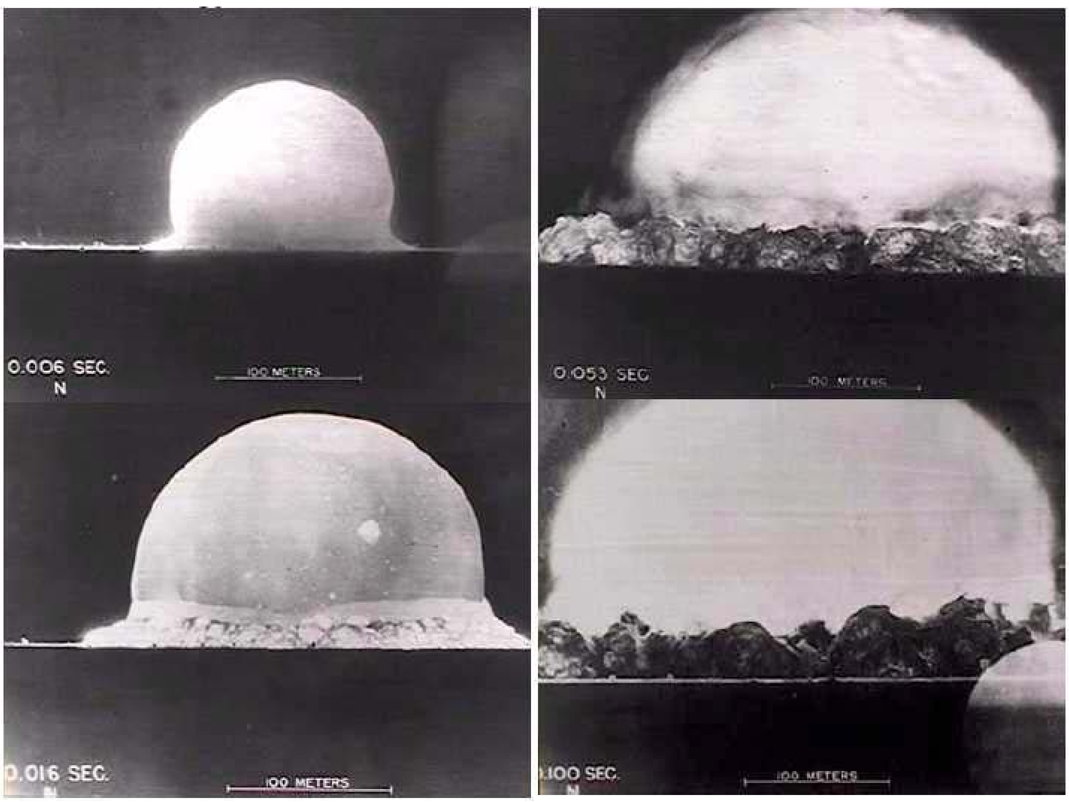
Fusion bomb - a much harder problem. First question: what nuclei are fused? deuterium + deuterium? Or, deuterium + tritium? How ignited? Nothing less than a fission bomb explosion could generate densities needed. About all I have to go on here are notes from Dr. Bethe's 1953 Cornell lectures on nucleosynthesis in stars.

Overall, situation hopeless. Too much I need to know is absent. JRO does not answer his phone.

Tuesday, 8:47 pm.
A colleague suggested an alternative approach. Call this the black box approach. Work only with what's available; assume little or nothing; work empirically. Ignore the detailed physics! The explosion could be fission or fusion powered. It could even be a chemical (TNT) explosion.

Do I have a choice? -- This is the only approach left.

Data -- these pictures clipped from a popular magazine. Found them in JRO's package. That's it for data!



An empirical, "black box" approach.

Data are just four measurements of the shockwave (fireball) radius (r) as a function of time (t). (My scientific apparatus: a straight-edge ruler). I measure by putting a ruler to the pictures.

time (sec)	**radius (meters)**
0.006	75
0.016	109
0.053	182
0.100	245

Explosive energy (E) released must govern radius. So come up with some equation in the form

$$r = f(E; t)$$

and fit it to the above data. Derive energy from best-fitting parameters.

It doesn't matter what kind of energy -- all that matters are the units of energy

$$\frac{\text{mass length length}}{\text{time time}}$$

But mass of what? Vexing question! Model assumptions are merely these:

- energy is released instantly (in much less than milliseconds) at a single location
- shockwave expands spherically (i.e., with simplest geometry)
- bomb explodes into an initially undisturbed, homogeneous medium with density ρ

Air density ρ is a second parameter. If energy and density enter the equation as a ratio, then mass cancels out! Thus,

$$\frac{E}{\rho} = \frac{\left(\dfrac{\text{mass length length}}{\text{time time}}\right)}{\left(\dfrac{\text{mass}}{\text{length length length}}\right)} = \frac{\text{length}^5}{\text{time}^2}$$

The right hand side of r = f(E; t) must include time as a variable. Because the units of the right and left hand sides must match, time must enter as t^2. So the empirical equation can only be

$$r = C\left(\frac{E}{\rho}t^2\right)^{\frac{1}{5}}$$

Constant C is a scaling factor. It probably should be there, but I don't know how to estimate it. For now, I will assume it to be equal to 1 and ignore it. Finally, I have something to go with!

To fit this power equation to the data, take the log of both sides. Thus,

$$\log(r) = \frac{1}{5}\left[\log\frac{E}{\rho} + 2\log(t)\right] = \frac{2}{5}\log(t) + \frac{1}{5}\log\frac{E}{\rho}$$

This is a straight line y = bx + a that can be fit to (log(t), log(r)) data points, where slope b is 2/5 and intercept a is (1/5) log (E/ρ).

<u>Model validity.</u> This model seems deceptively simple. It has the right behavior, thought. At any arbitrary time point, the radius is proportional to explosive energy and inversely proportional to density. (More density, more work done by shockwave as it expands, so less expansion). Model should have good dynamic range over energy. A 32-fold increase in E merely doubles r. Two conditions where the model breaks down and gives wrong results are

1. Explosion into a near vacuum. As density is reduced, shockwave velocity grows arbitrarily large, eventually exceeding the speed of light.

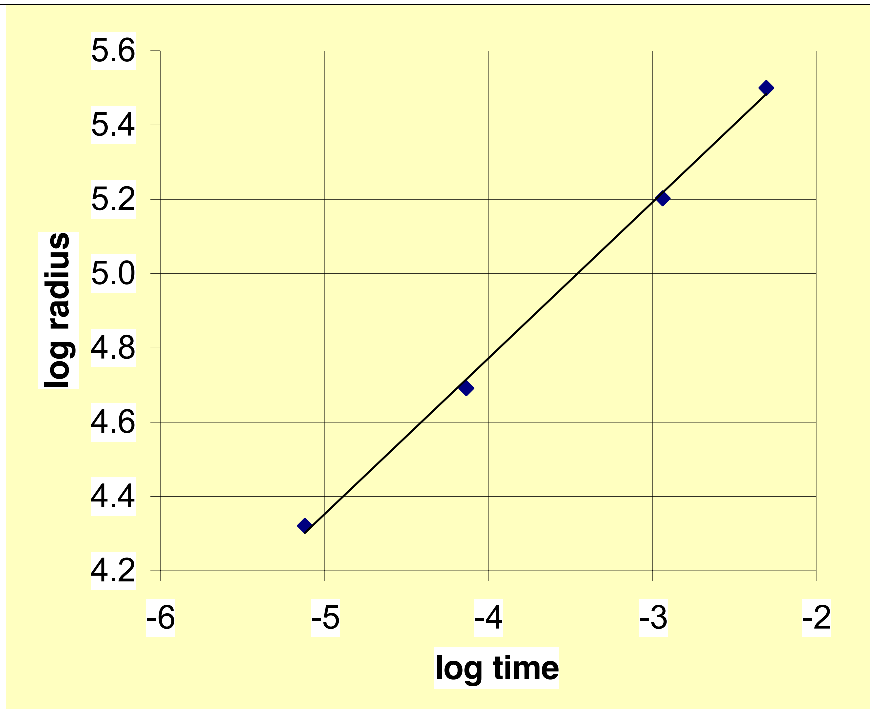
$$\lim_{\rho \to 0} \left(\frac{E}{\rho}t^2\right)^{\frac{1}{5}}_{t = t\text{fixed}} \to \infty$$

However, on the surface of the earth, air density is sizable and significant.

2. Shockwave dissipation is not predicted. Instead, the model says the shockwave will grow boundlessly.

$$\lim_{t \to \infty} \left(\frac{E}{\rho}t^2\right)^{\frac{1}{5}} \to \infty$$

However, my data concerns time only up to 100 milliseconds. So neither of these breakdown conditions are relevant to the data I am working with.



Least Squares Straight Line Fit

	log time	log radius			best fitting	
	x	**y**	**x2**	**xy**	**y**	**error**
	-5.12	4.32	26.17	-22.11	4.30	0.02
	-4.14	4.69	17.10	-19.40	4.71	-0.02
	-2.94	5.20	8.63	-15.28	5.22	-0.01
	-2.30	5.50	5.30	-12.67	5.48	0.02
sums	-14.49	19.72	57.20	-69.46		

N = number of data points =	4	
delta = N * sum(x2) - sum(x)2 =	18.82	
a = (sum(x2) * sum(y) - sum(x)*sum(xy)) / delta =	6.45	(intercept)
b = (N*sum(xy) - sum(x)*sum(y)) / delta =	0.419	(slope)

Best fitting line: y = bx + a = 0.419x + 6.45

Tuesday, 11:13 pm

Assessing the fit of the model to data.

Two observations suggest the model could be right. First, that a straight line fits well to (log(t), log(r)) data points is powerful evidence that the underlying equation must have the form

$$r = c\,t^{p}$$

where c and P are constants. So we have a power equation, not, for instance, an exponential equation. Second, the best fitting slope is 0.419, close to the theoretically expected value of 0.40. Thus the model is consistent with an exponent of 2/5.

Estimation of energy from the intercept term.

The fit yields this

$$\frac{1}{5}\log\frac{E}{\rho} = 6.45$$

meaning that

$$E = \rho\, e^{6.45*5} = 1.05\,(1.01\,x\,10^{14}) = 1.065x10^{14}\ J = 23\,Kt$$

So yield is 23 kilotons TNT, plus or minus about 10 kilotons.

Parameters used

- ρ (rho) is the density of air and is approximately 1.2 kg per cubic meter at sea level. (*Handbook of Chemistry and Physics 1948*). Data, I assume, are the New Mexico Trinity bomb test from 1945. Elevation there is about 5000 feet, so air density is less, roughly 1.0 to 1.1 kg/m^3. I'll use 1.05.

- Combustion energy of TNT is 4600 Joules / gram. One kiloton TNT is one billion grams TNT. Thus one kiloton (Kt) of TNT equals 4.6 x 10^{12} J of explosive energy.

Error bounds. I have no formal method of estimating error, so I will have to guess intuitively. I put 10 Kt of uncertainty on each side of the estimate.

11:47 pm -- Done, QED. This goes to JRO tomorrow.