APh161: The Physics of Biological Structure and Function
Homework 5
Due Date: Thursday, Mar. 1, 2007

"Their drills are bloodless battles and their battles are bloody drills." - Josephus

Reading: Read chap. 8 of PBOC and write a referee report in the usual way. Please remember as you are doing this that this is a huge help to us as we try to improve the book for its eventual publication. We appreciate it very much and it is a chance for you to construct a concrete written argument.

1. FRAP and diffusion.

In class I discussed the problem of two-dimensional FRAP. In this problem, I want you to carry out a full derivation of the concentration as a function of position and time after photobleaching a cell of radius 25 microns with a "hole" of radius 2 microns. (Looking at the treatment of the one-dimensional version of this problem in chap. 12 of PBOC will be helpful.) For simplicity, ignore the presence of a nucleus, think of the cell as a perfect circle and imagine the photobleached region as a circle at the center of the circular cell. Consider an initial concentration c_0 of the fluorescent molecule of interest which is uniformly distributed throughout the cell. How many molecules of the fluorescent molecule are there - write an equation that gives this number? Before doing any calculations, explain what the final concentration (c_∞) will be after full relaxation and the system has returned to equilibrium. You may assume that once a molecule has been photobleached it is effectively dead and can be forgotten. Your goal now is to compute the recovery curve. What this means is that you need to work out how many fluorescent molecules are in the photobleached region as a function of time. Make graphs for the case where the photobleached region is centered about the origin. Make sure when you make your plots you use reasonable values for the diffusion constant - justify your choice. One of the uses of the FRAP technique is to determine the diffusion constant of various molecules within the cytoplasm of cells. Discuss how that might work on the basis of the derivation you have given here. To do this problem you will need the table of zeros of the first derivative of $J_0(x)$ given in the file attached to the homework. Make sure you explain exactly what you are doing and what your results mean. Also, I want you to plot the result for different number of terms kept in the Bessel series. Try it for $N = 10, 20$ and 30 and compare the results. Comment on the goodness of the fit.

2. Gaussian Chain.

In class, I gave a heuristic "derivation" of the model of the Gaussian chain starting from the binomial distribution for chain configurations. Carry out a complete derivation of the Gaussian chain in one dimension by using the Stirling approximation as discussed in class. Make sure to explain how you turn this into a probability *density* and explain all of your steps. When you are done, plot the distribution of end-end distances for DNA of length 5 Kuhn lengths and 50 Kuhn lengths. Compare the discrete and continuous distributions. Using the distribution, compute the "size" of the polymer by evaluating $< x^2 >$ for the one-dimensional chain. Explain your result and compare it to the intuitive derivation given in class.