APh161: Physical Biology of the Cell
Homework 3
Due Date: Thursday, January 28, 2010

"Their drills are bloodless battles and their battles are bloody drills." - Josephus

Comments from RP to Class:

This third homework pushes you to develop some facility with binding problems. This kind of analysis will be crucial for much of what we do for the rest of the course, so developing familiarity with these problems is crucial. In addition, we work a little harder to understand how to think of DNA from the polymer physics perspective.

1. Free vs Total Ligands and Binding.

Do problem 6.3 of PBoC.

2. Binding Problems for Distinguishable Ligands.

Do problem 6.4 of PBoC.

3. Deriving the Canonical Distribution and Using it for the Problem of a Dishonest Die.

Read section 6.1.5 of PBoC and rederive and explain in your own way eqns. 6.47 and 6.63. Once you have read this section, do problem 6.8 of PBoC.

4. Binding of Dimoglobin.

Do problem 7.3 of PBoC. In addition, reproduce fig. 7.14. In reproducing the two figures, make sure you explain what the curves demonstrate (i.e. what do we learn about the binding problem from such curves?).

5. Random Walks and Polymers Revisited.

In this problem you will derive a continuum equation for the statistics of polymer chains in three dimensions and then derive the corresponding

"Green function". This Gaussian model is one of the most important results in physics and comes in handy not just for thinking about polymers, but also all sorts of interesting diffusive processes. In the context of this course, this thinking will have impact on the way we think about tethered-ligand receptor pairs in proteins such as N-Wasp and its synthetic analogs as well as when we think about conformations of DNA.

NOTE: there are some parts of this problem where you can "look up" part of the answer in PBoC. Obviously, my goal here is for you to make the arguments in your own way and using your own words. For example, last week, some people did the "fidelity" problem using the grand canonical distribution which is different than the way we did it in the book. That is exactly the kind of "putting your own spin" on the problems that I am after.

(a) Derive the equation for the one-dimensional probability distribution function by doing problem 8.4 of PBoC.

(b) In three-dimensions the relevant governing equation is

$$\frac{\partial p(\mathbf{r}, N)}{\partial N} = \frac{a^2}{6} \nabla^2 p(\mathbf{r}, N). \tag{1}$$

Using the results of part (a), make a heuristic argument as to why the equation has this form in 3D. Then, using the Fourier transform convention that

$$\tilde{p}(\mathbf{k}, N) = \int d^3 r \, e^{i\mathbf{k}\cdot\mathbf{r}} p(\mathbf{r}, N) \tag{2}$$

and

$$p(\mathbf{r}, N) = \frac{1}{(2\pi)^3} \int d^3 k \, e^{-i\mathbf{k}\cdot\mathbf{r}} \tilde{p}(\mathbf{k}, N), \tag{3}$$

Fourier transform the governing equation and solve for $\tilde{p}(\mathbf{k}, N)$ using the condition that $p(\mathbf{r}, 0) = \delta(\mathbf{r})$. Then compute the inverse Fourier transform to obtain the Green function $p(\mathbf{r}, N)$. Here when you do the inverse transform, I expecting you to do this analytically by completing the square, for example.

(c) Demonstrate that the probability distribution is normalized by showing that $\int_{\text{all space}} p(\mathbf{r}, N) d^3 \mathbf{r} = 1$.

(d) If the polymer starts at the origin, what is the probability density that it will end between r and $r + dr$ from the origin? Make a plot of this probability density.

(e) Finally, use your distribution to work out the probability of DNA loop formation (assuming that DNA is a Gaussian chain) by computing the probability that the end of the chain is within a small volume around the origin. How does this looping probability scale with the number of monomers in the chain?