BE/APh161: Physical Biology of the Cell
Homework 3
Due Date: Wednesday, January 25, 2012

"Habits are like cobwebs, then cables." - Spanish proverb - applies just as much to good habits as to bad!

Referee report Write a referee report on the two vignettes handed out with this homework, both of which have to do with photosynthesis.

1. The size of atoms.

In this problem, you will build on the calculations that I did in class to characterize the typical energy scales for electrons confined in boxes of different sizes. The goal here is to make a crude order of magnitude estimate of the size of an atom by examining the "competition" between the electrostatic potential between the proton in the nucleus of the atom and the electron, on the one hand, and the kinetic energy cost of confinement, on the other. To do this, write a total energy of a toy model of a hydrogen atom as a function of the parameter a which is of the form

$$E_{tot}(a) = E_{\text{kinetic energy}}(a) + E_{\text{electrostatic}}(a). \tag{1}$$

The confinement piece is precisely what I calculated in class and you should rederive it and explain how it scales with a. Do this by using the Heisenberg uncertainty principle. Then, write the interaction energy between the proton and the electron to obtain $E_{\text{electrostatic}}(a)$. Next, minimize with respect to the unknown atomic size a and find an expression for the size of the atom as a function of key parameters such as $\hbar$, m and e. How does your resulting expression compare to the Bohr radius?

2. A two-dimensional electron gas model for a "porphyrin" ring.

In this problem, we are going to use the same electron gas ideas developed in class to see whether we can construct a toy model of a porphyrin ring.

(a) Once again, we are going to make a crude model by treating the porphyrin ring structure as a circular box of radius R. Using the schematic in Fig. 1 and the facts that carbon-carbon single bonds are about 1.3 Å and double bonds about 1.5 Å, estimate the area of this circular box, and from that the radius R.

(b) Solve the Schrodinger equation for such a ring and find expressions for the energy eigenvalues as we did for the one-dimensional model in class. Then, assuming that each C and N atom in the ring donates one electron to the pi-orbital system of the ring, work out the excitation energy (and corresponding wavelength) as a function of the size of the ring.

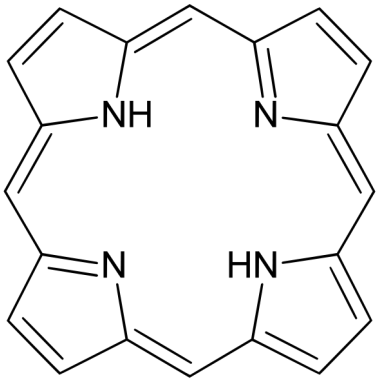

Figure 1: Structure of the simplest porphyrin, porphine.

NOTE: on part (b) you will have to separate variables and use the fact that the radial equation is a Bessel equation. Be careful and make sure you impose the boundary conditions correctly (i.e. vanishing of the wavefunction at the boundary). You can use Mathematica, Matlab or a handbook to find the zeros of the Bessel functions which you will need in order to find the energy eigenvalues. We will also post a list of the zeros of the Bessel functions.

3. The Story of Photosynthesis

Write a one-page *Scientific American*-style introduction to the biology, chemistry and physics of photosynthesis. This is a serious assignment. What I am after here is something that explains what photosynthesis is and how it works. If you can find a way to weave the interdisciplinary nature of the topic into your writeup, that is even better. Send the result in PDF form (no word documents!) to me and both TAs.