BE/APh161: Physical Biology of the Cell
Homework 4
Due Date: Wednesday, February 1, 2012

"Shun idleness. It is a rust that attaches itself to the most brilliant metals."
- Voltaire

1. Seeing the North star.

Polaris has been known to generations of northern hemisphere navigators as a tool for finding latitude by simple geometrical measurements with a sextant. How much light actually reaches our eyes from a star like Polaris? Given that the luminosity of Polaris is roughly 2000 times that of the sun (for your simple estimates you can use that the sun has a flux 1000 W/m^2 at the Earth) and that it is at a distance of 430 light years from Earth, work out the power output of Polaris, the number of photons crossing the pupil of your eye each second coming from this famous star and the mean spacing between these photons. What is the mean rate of arrival of photons to a single cone cell?

(Problem adapted from a beautiful discussion in the book of R. W. Rodieck, though we do not agree with all of his numbers.)

2. Fleshing out the vision thing.

(A) In class, I wrote an integral for the Fraunhofer diffraction from a circular aperture. Justify this integral in your own words being sure to explain why I was referring to stopwatches and how this captures the idea of different phases for different rays going out to the detection point. The key points you are trying to get across in this part of the problem are the underlying physical basis for the diffraction from the circular aperture and making sure that the mathematical formula is correct.

(B) Go through all of the messy steps of doing the integral from part (A) to arrive at the simple formula for the angular resolution, namely,

$$\sin\,\theta = 1.22\frac{\lambda}{D}. \tag{1}$$

What this amounts to is using the results from part (A) to carry out the relevant integral and then interpreting the result in terms of the shape of the intensity profile. In particular, the image plane is at a distance Z from the aperture and the angular resolution is given by

$$\sin\,\theta = \frac{X}{Z}, \tag{2}$$

where X is the distance between the origin and the point of interest on the X-axis on the image plane.

Some useful mathematical facts that will come in handy for doing this problem include

$$\int_0^{2\pi} d\theta e^{ix\cos\theta} = 2\pi J_0(x) \tag{3}$$

and

$$\int xJ_0(x)dx = xJ_1(x). \tag{4}$$

What I have in mind in this part is that you carry through the entire derivation that was sketched in class, explaining the approximations that you make and the overall logic. Then, I want to make sure you explain how this tells us something about the resolution of an optical instrument (including the eye).

(C) Plot the sum of two of the intensity curves you calculated in part (B) for several different distances between their centers and use this to explain the argument about the diffraction limit and how objects can be resolved.

(D) Do the following experiment. Draw two lines, first, 1 mm apart and second, 2 mm apart. Tape the paper with these lines to the wall and now walk backwards until you reach a point where you can no longer distinguish the lines as being distinct. What is their angular separation in the two cases? How does this compare to the calculated limit?

3. Vision: Your Turn.

Write a short Scientific American style article (no more than one page) that explains vision in vertebrates from the arrival of a single photon to the propagation of the downstream electrical signal that reflects the arrival of the photon. As with photosynthesis, I am able to only do little bits and pieces of the subject of vision from a quantitative perspective and the point of these little write ups is to get you to fill in some of the gaps that I leave and make sure you have a sense of the overall process. Clearly one of the very best resources for you to do some background reading is: http://webvision.med.utah.edu/book/. There is also an excellent book by Dowling called "The Retina". Some of the key points that I want you to touch on include the anatomy of the cells that detect light, the molecules that are influenced by light and the signal transduction cascade that occurs once retinal has been excited.