BE/APh161: Physical Biology of the Cell
Homework 5
Due Date: Wednesday, February 8, 2012

1. Fleshing out the vision thing.

RP to class: we assigned this last week, but I did not get to it because I decided to change the logic of the lectures. Now this is a part of homework 5.

(A) In class, I wrote an integral for the Fraunhofer diffraction from a circular aperture. Justify this integral in your own words being sure to explain why I was referring to stopwatches and how this captures the idea of different phases for different rays going out to the detection point. The key points you are trying to get across in this part of the problem are the underlying physical basis for the diffraction from the circular aperture and making sure that the mathematical formula is correct.

(B) Go through all of the messy steps of doing the integral from part (A) to arrive at the simple formula for the angular resolution, namely,

$$\sin\,\theta = 1.22\frac{\lambda}{D}. \tag{1}$$

What this amounts to is using the results from part (A) to carry out the relevant integral and then interpreting the result in terms of the shape of the intensity profile. In particular, the image plane is at a distance Z from the aperture and the angular resolution is given by

$$\sin\,\theta = \frac{X}{Z}, \tag{2}$$

where X is the distance between the origin and the point of interest on the X-axis on the image plane.

Some useful mathematical facts that will come in handy for doing this problem include

$$\int_0^{2\pi} d\theta e^{ix\cos\theta} = 2\pi J_0(x) \tag{3}$$

and

$$\int xJ_0(x)dx = xJ_1(x). \tag{4}$$

What I have in mind in this part is that you carry through the entire derivation that was sketched in class, explaining the approximations that you make and the overall logic. Then, I want to make sure you explain how this tells us something about the resolution of an optical instrument (including the eye).

(C) Plot the sum of two of the intensity curves you calculated in part (B) for several different distances between their centers and use this to explain the argument about the diffraction limit and how objects can be resolved.

(D) Do the following experiment. Draw two lines, first, 1 mm apart and second, 2 mm apart. Tape the paper with these lines to the wall and now walk backwards until you reach a point where you can no longer distinguish the lines as being distinct. What is their angular separation in the two cases? How does this compare to the calculated limit?

2. Ion channel properties.

Work out problem 17.5 of PBoC2.

3. Diffusion and the Fourier transform.

Work out problem 13.2 of PBoC.