BE/APh161: Physical Biology of the Cell
Homework 4
Due Date: Wednesday, February 4, 2015

"You can observe a lot just by watching." - Yogi Berra

Useful reading can be found in chaps. 6 and 19 of PBoC.

1. Optical Traps.

Do problem 6.2 of PBoC2 - this problem is about the statistical mechanics of an optical trap and will help you develop facility in using statistical mechanics. Make sure you explain how the ideas discussed in this problem can be used to calibrate an optical trap (what does that even mean?). Please be thoughtful and careful about the fact that the partition function requires you to "sum" over an infinite set of states (i.e. the continuous set of positions of the bead in the trap).

2. Poisson promoter.

(a) Use the thermodynamic model of gene expression to derive the probability that the promoter will be occupied by RNA polymerase for the case of an unregulated promoter (i.e. there are no transcription factors that alter polymerase binding). This basically repeats things I considered in class and the main point in this part of the problem is for you to explain all of the conceptual and mathematical steps and just make sure you see how to do it for yourself.

(b) Building on the theme of the unregulated promoter, do problem 19.10 of PBoC2. Here we use rate equations to consider the behavior of the unregulated promoter with the additional twist that we include the replication of the genome over the cell cycle.

3. Simple repression.

(a) Repeat problem 2a for the case of simple repression, where the attachment of a repressor to its binding site prevents polymerase from binding. Show

that the probability of RNAP binding can be written as

$$p_{bound} = \frac{1}{1 + \frac{N_{NS}}{PF_{reg}}e^{\beta\Delta\epsilon_p}},$$ (1)

and find an expression for $F_{reg} = (\text{Repression})^{-1}$ for a weak promoter. Make a plot of repression (inverse of fold-change) as a function of the number of repressors assuming that the binding energy of repressor to DNA takes the values -10 k_BT and -15 k_BT. Give an intuitive explanation of how the repression scales with number of repressors and with the binding energy.

(b) Now that you know how to derive the repression, let's try it out as the basis for explaining some very interesting real world data. In particular, do problem 19.2, part (a) of PBoC2. Make sure you explain how you obtained the binding energies.