Focusing and Collimating

Optical Ray Tracing

An introduction to the use of lenses to solve optical applications can begin with the elements of ray tracing. Figure 1 demonstrates an elementary ray trace showing the formation of an image, using an ideal thin lens. The object height is y_1 at a distance s_1 from an ideal thin lens of focal length f. The lens produces an image of height y_2 at a distance s_2 on the far side of the lens.

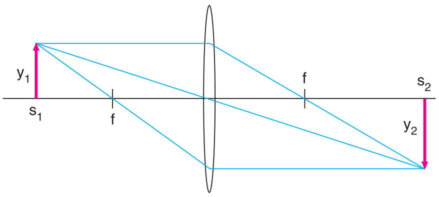

Figure 1

By **ideal thin lens**, we mean a lens whose thickness is sufficiently small that it does not contribute to its focal length. In this case, the change in the path of a beam going through the lens can be considered to be instantaneous at the center of the lens, as shown in the figure. In the applications described here, we will assume that we are working with ideally thin lenses. This should be sufficient for an introductory discussion. Consideration of aberrations and thick-lens effects will not be included here.

Three rays are shown in Figure 1. Any two of these three rays fully determine the size and position of the image. One ray emanates from the object parallel to the optical axis of the lens. The lens refracts this beam through the optical axis at a distance f on the far side of the lens. A second ray passes through the optical axis at a distance f in front of the lens. This ray is then refracted into a path parallel to the optical axis on the far side of the lens. The third ray passes through the center of the lens. Since the surfaces of the lens are normal to the optical axis and the lens is very thin, the deflection of this ray is negligible as it passes through the lens.

In addition to the assumption of an ideally thin lens, we also work in the paraxial approximation. That is, angles are small and we can substitute θ in place of $\sin \theta$.

Magnification

We can use basic geometry to look at the magnification of a lens. In Figure 2, we have the same ray tracing figure with some particular line segments highlighted. The ray through the center of the lens and the optical axis intersect at an angle ϕ. Recall that the opposite angles of two intersecting lines are equal. Therefore, we have two similar triangles. Taking the ratios of the sides, we have

$\phi = y_1/s_1 = y_2/s_2$

This can then be rearranged to give

$y_2/y_1 = s_2/s_1 = M.$

The quantity M is the **magnification** of the object by the lens. The magnification is the ratio of the image size to the object size, and it is also the ratio of the image distance to the object distance.

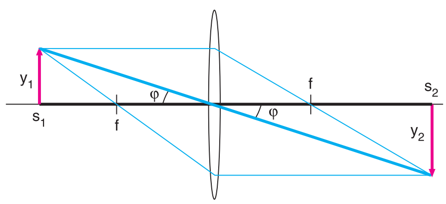

Figure 2

This puts a fundamental limitation on the geometry of an optics system. If an optical system of a given size is to produce a particular magnification, then there is only one lens position that will satisfy that requirement. On the other hand, a big advantage is that one does not need to make a direct measurement of the object and image sizes to know the magnification; it is determined by the geometry of the imaging system itself.

Gaussian Lens Equation

Let's now go back to our ray tracing diagram and look at one more set of line segments. In Figure 3, we look at the optical axis and the ray through the front focus. Again looking at similar triangles sharing a common vertex and, now, angle η, we have $y_2/f = y_1/(s_1-f)$.

Rearranging and using our definition of magnification, we find

$y_2/y_1 = s_2/s_1 = f/(s_1-f).$

Rearranging one more time, we finally arrive at

$1/f = 1/s_1 + 1/s_2.$

This is the **Gaussian lens equation**. This equation provides the fundamental relation between the focal length of the lens and the size of the optical system. A specification of the required magnification and the Gaussian lens equation form a system of two equations with three unknowns: f, s_1, and s_2. The addition of one final condition will fix these three variables in an application.

This additional condition is often the focal length of the lens, f, or the size of the object to image distance, in which case the sum of $s_1 + s_2$ is given by the size constraint of the system. In either case, all three variables are then fully determined.

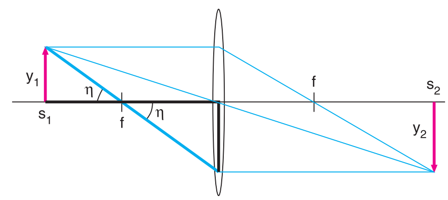

Figure 3

Optical Invariant

Now we are ready to look at what happens to an arbitrary ray that passes through the optical system. Figure 4 shows such a ray. In this figure, we have chosen the maximal ray, that is, the ray that makes the maximal angle with the optical axis as it leaves the object, passing through the lens at its maximum clear aperture. This choice makes it easier, of course, to visualize what is happening in the system, but this maximal ray is also the one that is of most importance in designing an application. While the figure is drawn in this fashion, the choice is completely arbitrary and the development shown here is true regardless of which ray is actually chosen.

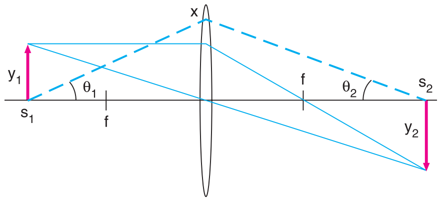

Figure 4

This arbitrary ray goes through the lens at a distance x from the optical axis. If we again apply some basic geometry, we have, using our definition of the magnification,

$\theta_1 = x/s_1$ and $\theta_2 = x/s_2 = (x/s_1)(y_1/y_2)$.

Rearranging, we arrive at

$y_2\theta_2 = y_1\theta_1$.

This is a fundamental law of optics. In any optical system comprising only lenses, the product of the image size and ray angle is a constant, or invariant, of the system. This is known as the **optical invariant**. The result is valid for any number of lenses, as could be verified by tracing the ray through a series of lenses. In some optics textbooks, this is also called the Lagrange Invariant or the Smith-Helmholz Invariant.

This is valid in the paraxial approximation in which we have been working. Also, this development assumes perfect, aberration-free lenses. The addition of aberrations to our consideration would mean the replacement of the equal sign by a greater-than-or-equal sign in the statement of the invariant. That is, aberrations could increase the product but nothing can make it decrease.

Application 1: Focusing a Collimated Laser Beam

As a first example, we look at a common application, the focusing of a laser beam to a small spot. The situation is shown in Figure 5. Here we have a laser beam, with radius y_1 and divergence θ_1 that is focused by a lens of focal length f. From the figure, we have $\theta_2 = y_1/f$. The optical invariant then tells us that we **must** have $y_2 = \theta_1 f$, because the product of radius and divergence angle must be constant.

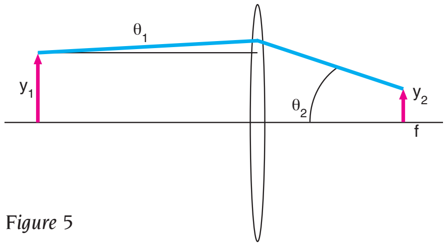

Figure 5

As a numerical example, let's look at the case of the output from a Newport R-31005 HeNe laser focused to a spot using a Newport KPX043 plano-convex lens. This laser has a beam diameter of 0.63 mm and a divergence of 1.3 mrad. Note that these are beam diameter and full divergence, so in the notation of our figure, $y_1 = 0.315$ mm and $\theta_1 = 0.65$ mrad. The KPX043 lens has a focal length of 25.4 mm. Thus, at the focused spot, we have a radius $\theta_1 f = 16.5$ µm. So, the diameter of the spot will be 33 µm.

This is a fundamental limitation on the minimum size of the focused spot in this application. We have already assumed a perfect, aberration-free lens. No improvement of the lens can yield any improvement in the spot size. The only way to make the spot size smaller is to use a lens of shorter focal length or expand the beam. If this is not possible because of a limitation in the geometry of the optical system, then this spot size is the smallest that could be achieved. In addition, diffraction may limit the spot to an even larger size (see Gaussian Beam Optics section beginning on page 484), but we are ignoring wave optics and only considering ray optics here.

Application 2: Collimating Light from a Point Source

Another common application is the collimation of light from a very small source, as shown in Figure 6. The problem is often stated in terms of collimating the output from a "point source." Unfortunately, nothing is ever a true point source and the size of the source must be included in any calculation. In figure 6, the point source has a radius of y_1 and has a maximum ray of angle θ_1. If we collimate the output from this source using a lens with focal length f, then the result will be a beam with a radius $y_2 = \theta_1 f$ and divergence angle $\theta_2 = y_1/f$. Note that, no matter what lens is used, the beam radius and beam divergence have a reciprocal relation. For example, to improve the collimation by a factor of two, you need to increase the beam diameter by a factor of two.

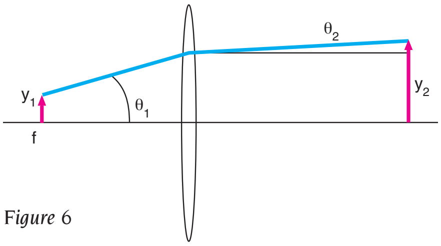

Figure 6

Since a common application is the collimation of the output from an optical fiber, let's use that for our numerical example. The Newport F-MBB fiber has a core diameter of 200 µm and a numerical aperture (NA) of 0.37. The radius y_1 of our source is then 100 µm. NA is defined in terms of the half-angle accepted by the fiber, so $\theta_1 = 0.37$. If we again use the KPX043, 25.4 mm focal length lens to collimate the output, we will have a beam with a radius of 9.4 mm and a half-angle divergence of 4 mrad. We are locked into a particular relation between the size and divergence of the beam. If we want a smaller beam, we must settle for a larger divergence. If we want the beam to remain collimated over a large distance, then we must accept a larger beam diameter in order to achieve this.

Application 3: Expanding a Laser Beam

It is often desirable to expand a laser beam. At least two lenses are necessary to accomplish this. In Figure 7, a laser beam of radius y_1 and divergence θ_1 is expanded by a negative lens with focal length $-f_1$. From Applications 1.1 and 1.2 we know $\theta_2 = y_1/|-f_1|$, and the optical invariant tells us that the radius of the virtual image formed by this lens is $y_2 = \theta_1|-f_1|$. This image is at the focal point of the lens, $s_2 = -f_1$, because a well-collimated laser yields $s_1 \sim \infty$, so from the Gaussian lens equation $s_2 = f$. Adding a second lens with a positive focal length f_2 and separating the two lenses by the sum of the two focal lengths $-f_1 + f_2$, results in a beam with a radius $y_3 = \theta_2 f_2$ and divergence angle $\theta_3 = y_2/f_2$.

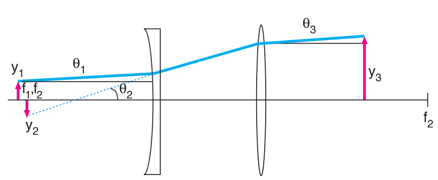

Figure 7

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The expansion ratio

$y_3/y_1 = \theta_2 f_2/\theta_2|-f_1| = f_2/|-f_1|$,

or the ratio of the focal lengths of the lenses. The expanded beam diameter

$2y_3 = 2\theta_2 f_2 = 2y_1 f_2/|-f_1|$.

The divergence angle of the resulting expanded beam

$\theta_3 = y_2/f_2 = \theta_1|-f_1|/f_2$

is reduced from the original divergence by a factor that is equal to the ratio of the focal lengths $|-f_1|/f_2$. So, to expand a laser beam by a factor of five we would select two lenses whose focal lengths differ by a factor of five, and the divergence angle of the expanded beam would be 1/5th the original divergence angle.

As an example, consider a Newport R-31005 HeNe laser with beam diameter 0.63 mm and a divergence of 1.3 mrad. Note that these are beam diameter and full divergence, so in the notation of our figure, $y_1 = 0.315$ mm and $\theta_1 = 0.65$ mrad. To expand this beam ten times while reducing the divergence by a factor of ten, we could select a plano-concave lens KPC043 with $f_1 = -25$ mm and a plano-convex lens KPX109 with $f_2 = 250$ mm. Since real lenses differ in some degree from thin lenses, the spacing between the pair of lenses is actually the sum of the back focal lengths $BFL_1 + BFL_2 = -26.64$ mm + 247.61 mm = 220.97 mm.

The expanded beam diameter

$$2y_3 = 2y_1 f_2/|-f_1| = 2(0.315 \text{ mm})(250 \text{ mm})/|-25 \text{ mm}| = 6.3 \text{ mm}.$$

The divergence angle

$$\theta_3 = \theta_1|-f_1|/f_2 = (0.65 \text{ mrad})|-25 \text{ mm}|/250 \text{ mm} = 0.065 \text{ mrad}.$$

For minimal aberrations, it is best to use a plano-concave lens for the negative lens and a plano-convex lens for the positive lens with the plano surfaces facing each other. To further reduce aberrations, only the central portion of the lens should be illuminated, so choosing oversized lenses is often a good idea. This style of beam expander is called Galilean. Two positive lenses can also be used in a Keplerian beam expander design, but this configuration is longer than the Galilean design.

Application 4: Focusing an Extended Source to a Small Spot

This application is one that will be approached as an imaging problem as opposed to the focusing and collimation problems of the previous applications. An example might be a situation where a fluorescing sample must be imaged with a CCD camera. The geometry of the application is shown in Figure 8. An extended source with a radius of y_1 is located at a distance s_1 from a lens of focal length f. The figure shows a ray incident upon the lens at a radius of R. We can take this radius R to be the maximal allowed ray, or clear aperture, of the lens.

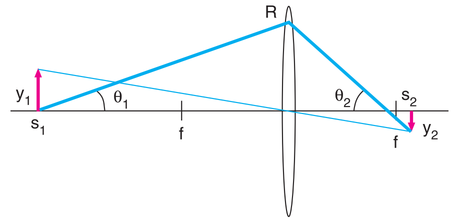

Figure 8

If s_1 is large, then s_2 will be close to f, from our Gaussian lens equation, so for the purposes of approximation we can take $\theta_2 \sim R/f$. Then from the optical invariant, we have

$y_2 = y_1\theta_1/\theta_2 = y_1(R/s_1)(f/R)$ or

$y_2 = 2y_1(R/s_1)f/\#$.

where f/2R = f/D is the f-number, f/#, of the lens. In order to make the image size smaller, we could make f/# smaller, but we are limited to f/# = 1 or so. That leaves us with the choice of decreasing R (smaller lens or aperture stop in front of the lens) or increasing s_1. However, if we do either of those, it will restrict the light gathered by the lens. If we either decrease R by a factor of two or increase s_1 by a factor of two, it would decrease the total light focused at s_2 by a factor of four due to the restriction of the solid angle subtended by the lens.

Fiber Optic Coupling

The problem of coupling light into an optical fiber is really two separate problems. In one case, we have the problem of coupling into multimode fibers, where the ray optics of the previous section can be used. In the other case, coupling into single-mode fibers, we have a fundamentally different problem. In this case, one must consider the problem of matching the mode of the incident laser light into the mode of the fiber. This cannot be done using the ray optics approach, but must be done using the concepts of Gaussian beam optics (see page 484).

Application 5: Coupling Laser Light into a Multimode Fiber

When we look at coupling light from a well-collimated laser beam into a multimode optical fiber, we return to the situation that was illustrated in Figure 5. The radius of the fiber core will be our y_2. We will have to make sure that the lens focuses to a spot size less than this parameter. An even more important restriction is that the angle from the lens to the fiber θ_2 must be less than the NA of the optical fiber.

Let's consider coupling the light from a Newport R-30990 HeNe laser into an F-MSD fiber. The laser has a beam diameter of 0.81 mm and divergence .0 mrad. The fiber has a core diameter of 50 µm and an NA of 0.20. Let's look at the coupling from the beam into the fiber when a Newport M-20X objective lens is used in an F-915 or F-915T fiber coupler.

The objective lens has an effective focal length of 9 mm. In this case, the focused beam will have a diameter of 9 µm and a maximal ray of angle 0.05, so both the