TECHNICAL REFERENCE AND FUNDAMENTAL APPLICATIONS

LENS SELECTION GUIDE

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Gaussian Beam Optics

The Gaussian is a radially symmetrical distribution whose electric field variation is given by the following equation :

$$E_S = E_0 \exp\left(-\frac{r^2}{\omega_0^2}\right)$$

Its Fourier Transform is also a Gaussian distribution. If we were to solve the Fresnel integral itself rather than the Fraunhofer approximation, we would find that a Gaussian source distribution remains Gaussian at every point along its path of propagation through the optical system. This makes it particularly easy to visualize the distribution of the fields at any point in the optical system. The intensity is also Gaussian:

$$I_S = \eta E_S E_S^* = \eta E_0 E_0^* \exp\left(-\frac{2r^2}{\omega_0^2}\right)$$

This relationship is much more than a mathematical curiosity, since it is now easy to find a light source with a Gaussian intensity distribution: the laser. Most lasers automatically oscillate with a Gaussian distribution of electrical field. The basic Gaussian may also take on some particular polynomial multipliers and still remain its own transform. These field distributions are known as higher-order transverse modes and are usually avoided by design in most practical lasers.

The Gaussian has no obvious boundaries to give it a characteristic dimension like the diameter of the circular aperture, so the definition of the size of a Gaussian is somewhat arbitrary. Figure 1 shows the Gaussian intensity distribution of a typical HeNe laser.

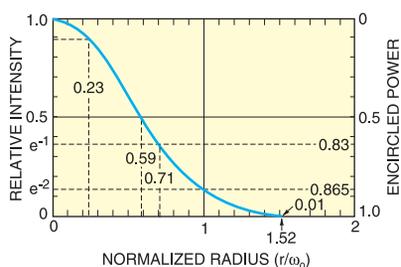$$I(r) = I_0 \exp\left(-\frac{2r^2}{\omega_0^2}\right)$$

Figure 1

The parameter ω_0, usually called the Gaussian beam radius, is the radius at which the intensity has decreased to $1/e^2$ or 0.135 of its axial, or peak value. Another point to note is the radius of half maximum, or 50% intensity, which is $0.59\omega_0$. At $2\omega_0$, or twice the Gaussian radius, the intensity is 0.0003 of its peak value, usually completely negligible.

The power contained within a radius r, P(r), is easily obtained by integrating the intensity distribution from 0 to r:

$$P(r) = P(\infty)\left[1 - \exp\left(\frac{-2r^2}{\omega_0^2}\right)\right]$$

When normalized to the total power of the beam, $P(\infty)$ in watts, the curve is the same as that for intensity, but with the ordinate inverted. Nearly 100% of the power is contained in a radius $r = 2\omega_0$. One-half the power is contained within $0.59\omega_0$, and only about 10% of the power is contained with $0.23\omega_0$, the radius at which the intensity has decreased by 10%. The total power, $P(\infty)$ in watts, is related to the on-axis intensity, I(0) (watts/m^2), by:

$$P(\infty) = \left(\frac{\pi\omega_0^2}{2}\right) I(0)$$

$$I(0) = P(\infty)\left(\frac{2}{\pi\omega_0^2}\right)$$

The on-axis intensity can be very high due to the small area of the beam.

Care should be taken in cutting off the beam with a very small aperture. The source distribution would no longer be Gaussian, and the far-field intensity distribution would develop zeros and other non-Gaussian features. However, if the aperture is at least three or four ω_0 in diameter, these effects are negligible.

Propagation of Gaussian beams through an optical system can be treated almost as simply as geometric optics. Because of the unique self-Fourier Transform characteristic of the Gaussian, we do not need an integral to describe the evolution of the intensity profile with distance. The transverse distribution intensity remains Gaussian at every point in the system; only the radius of the Gaussian and the

radius of curvature of the wavefront change. Imagine that we somehow create a coherent light beam with a Gaussian distribution and a plane wavefront at a position x=0. The beam size and wavefront curvature will then vary with x as shown in Figure 2.
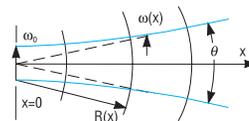

Figure 2

The beam size will increase, slowly at first, then faster, eventually increasing proportionally to x. The wavefront radius of curvature, which was infinite at x = 0, will become finite and initially decrease with x. At some point it will reach a minimum value, then increase with larger x, eventually becoming proportional to x. The equations describing the Gaussian beam radius w(x) and wavefront radius of curvature R(x) are:

$$\omega^2(x) = \omega_0^2\left[1 + \left(\frac{\lambda x}{\pi\omega_0^2}\right)^2\right]$$

$$R(x) = x\left[1 + \left(\frac{\pi\omega_0^2}{\lambda x}\right)^2\right]$$

where ω_0 is the beam radius at x = 0 and λ is the wavelength. The entire beam behavior is specified by these two parameters, and because they occur in the same combination in both equations, they are often merged into a single parameter, x_R, the Rayleigh range:

$$x_R = \frac{\pi\omega_0^2}{\lambda}$$

In fact, it is at x = x_R that R has its minimum value.

Note that these equations are also valid for negative values of x. We only imagined that the source of the beam was at x = 0; we could have created the same beam by creating a larger Gaussian beam with a negative wavefront curvature at some x < 0. This we can easily do with a lens, as shown in Figure 3.
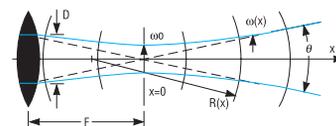

Figure 3

The input to the lens is a Gaussian with diameter D and a wavefront radius of curvature which, when modified by the lens, will be R(x) given by the equation above with the lens located at -x from the beam waist at x = 0. That input Gaussian will also have a beam waist position and size associated with it. Thus we can generalize the law of propagation of a Gaussian through even a complicated optical system.

In the free space between lenses, mirrors and other optical elements, the position of the beam waist and the waist diameter completely describe the beam. When a beam passes through a lens, mirror, or dielectric interface, the diameter is unchanged but the wavefront curvature is changed, resulting in new values of waist position and waist diameter on the output side of the interface.

These equations, with input values for ω and R, allow the tracing of a Gaussian beam through any optical system with some restrictions: optical surfaces need to be spherical and with not-too-short focal lengths, so that beams do not change diameter too fast. These are exactly the analog of the paraxial restrictions used to simplify geometric optical propagation.

It turns out that we can put these laws in a form as convenient as the ABCD matrices used for geometric ray tracing. But there is a difference: ω(x) and R(x) do not transform in matrix fashion as r and u do for ray tracing; rather, they transform via a complex bi-linear transformation:

$$q_{out} = \frac{\left[q_{in}\, A + B \right]}{\left[q_{in}\, C + D \right]}$$

where the quantity q is a complex composite of ω and R:

$$\frac{1}{q(x)} = \frac{1}{R(x)} - \frac{i\lambda}{\pi w(x)^2}$$

We can see from the expression for q that at a beam waist (R = ∞ and ω = ω$_0$), q is pure imaginary and equals ix$_R$. If we know where one beam waist is and its size, we can calculate q there and then use the bilinear ABCD relation to find q anywhere

else. To determine the size and wavefront curvature of the beam everywhere in the system, you would use the ABCD values for each element of the system and trace q through them via successive bilinear transformations. But if you only wanted the overall transformation of q, you could multiply the elemental ABCD values in matrix form, just as is done in geometric optics, to find the overall ABCD values for the system, then apply the bilinear transform. For more information about Gaussian beams, see Anthony E. Siegman's book, *Lasers* (University Science Books, 1986).

Fortunately, simple approximations for spot size and depth of focus can still be used in most optical systems to select pinhole diameters, couple light into fibers, or compute laser intensities. Only when f-numbers are large should the full Gaussian equations be needed.

At large distances from a beam waist, the beam appears to diverge as a spherical wave from a point source located at the center of the waist. Note that "large" distances mean where x»x$_R$ and are typically very manageable considering the small area of most laser beams. The diverging beam has a full angular width θ (again, defined by 1/e^2 points):

$$\theta = \frac{4\lambda}{2\pi w_0}$$

We have invoked the approximation tanθ ≈ θ since the angles are small. Since the origin can be approximated by a point source, θ is given by geometrical optics as the diameter illuminated on the lens, D, divided by the focal length of the lens.

$$\theta \approx \frac{D}{F} = (f/\#)^{-1}$$

where f/# is the photographic f-number of the lens.

Equating these two expressions allows us to find the beam waist diameter in terms of the input beam parameters (with some restrictions that will be discussed later):

$$2w_0 = \left(\frac{4\lambda}{\pi} \right)\left(\frac{F}{D} \right)$$

We can also find the depth of focus from the formulas above. If we define the

depth of focus (somewhat arbitrarily) as the distance between the values of x where the beam is √2 times larger than it is at the beam waist, then using the equation for ω(x) we can determine the depth of focus:

$$DOF = \left(\frac{8\lambda}{\pi} \right)\left(\frac{F}{D} \right)^2$$

Using these relations, we can make simple calculations for optical systems employing Gaussian beams. For example, suppose that we use a 10 mm focal length lens to focus the collimated output of a helium-neon laser (632.8 nm) that has a 1 mm diameter beam. The diameter of the focal spot will be:

$$\left(\frac{4}{\pi} \right)\left(632,8\ nm \right)\left(\frac{10\ mm}{1\ mm} \right),$$

or about 8 μm. The depth of focus for the beam is then:

$$\left(\frac{8}{\pi} \right)\left(632,8\ nm \right)\left(\frac{10\ mm}{1\ mm} \right)^2,$$

or about 160 μm. If we were to change the focal length of the lens in this example to 100 mm, the focal spot size would increase 10 times to 80 μm, or 8% of the original beam diameter. The depth of focus would increase 100 times to 16 mm. However, suppose we increase the focal length of the lens to 2,000 mm. The "focal spot size" given by our simple equation would be 200 times larger, or 1.6 mm, 60% larger than the original beam! Obviously, something is wrong. The trouble is not with the equations giving ω(x) and R(x), but with the assumption that the beam waist occurs at the focal distance from the lens. For weakly focused systems, the beam waist does not occur at the focal length. In fact, the position of the beam waist changes contrary to what we would expect in geometric optics: the waist moves toward the lens as the focal length of the lens is increased. However, we could easily believe the limiting case of this behavior by noting that a lens of infinite focal length such as a flat piece of glass placed at the beam waist of a collimated beam will produce a new beam waist not at infinity, but at the position of the glass itself.

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