Bi 1: The Great Ideas of Biology
Homework 1
Due Date: Thursday, April 13, 2023

Everything -
a bumptious, stuck-up word.
It should be written in quotes.
It pretends to miss nothing,
to gather, hold, contain and have.
While all the while it's just
a thread pulled out of a giant tangle.

Wislawa Szymborska

1. The Cost of Getting Around

This first problem set involves a number of challenges in order-of-magnitude thinking. Remember that we are making "street-fighting estimates", where the goal is to do simple arithmetic following the guidelines that all numbers take the values 1, few (f) or 10; $f \times f = 10$; and $\frac{1}{f} = \frac{f}{10}$. We also have tools like the geometric mean to use when confronted by quantities we don't know. Please do not provide estimates with multiple "significant" digits; this misses the point, which is to do sanity checks and to develop intuition. Be thoughtful about what you know and what you don't know. If you have some source for your numbers, please cite it; but in general, we think these exercises will be more valuable if you do them without looking things up.

Drag Forces on Whales

As any swimmer knows, if you push off the wall and simply glide, after less than half a pool length you will come to a stop: the inevitable consequence of the drag force due to the water you are swimming through. Whether birds and planes moving through air, or whales or boats moving through water, drag forces are a critical part of the story of how much fuel needs to be consumed to travel a given distance in a fluid medium. In class, we discussed the migration of whales, noting that once humpacks leave Alaska for Hawaii, they stop feeding. As their body fat is consumed to fuel this long distance

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swim, they experience a significant loss in body mass. In this problem, we try to estimate the magnitude of these effects; later on, we use the results to understand animal motion and metabolism more broadly.

| PARAMETERS INFLUENCING THE DRAG FORCE ON THE WHALE | RELEVANT PARAMETERS | ESTIMATING THE DRAG FORCE BY DIMENSIONAL ANALYSIS |

1 density of water ρ, units of $\frac{M}{L^3}$

water

what is the drag force, F_{drag} on a swimming whale?

2 speed of the whale v, units of $\frac{L}{T}$

v

3 size of the whale ℓ, units of L

ℓ

unknown exponents

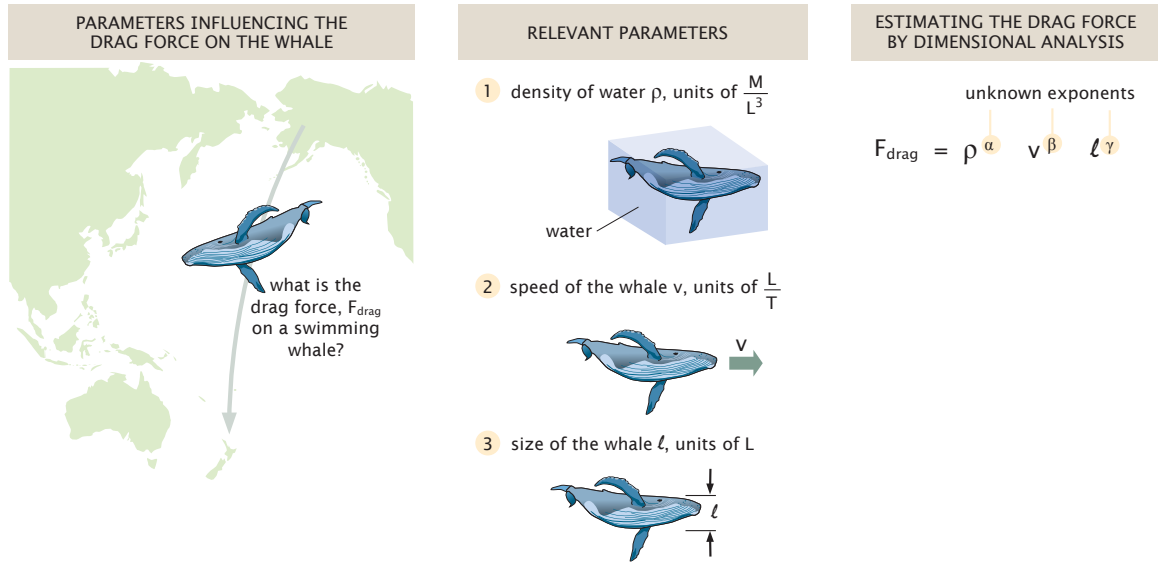$$F_{drag} = \rho^{\alpha} \; v^{\beta} \; \ell^{\gamma}$$

Figure 1: Humpback whales travel thousands of kilometers between where they eat in Alaska and their breeding grounds in Hawaii. To estimate the drag force associated with whale swimming, we hypothesize that it scales with the density of the water, the speed of the whale, and the size of the whale. In a scaling estimate, we seek choices for the parameters α, β and γ that are dimensionally consistent. These ideas can be formalized in the form of the Buckingham Pi theorem, although here we take a "scratch and sniff" approach instead.

Question 1a

Using the concept highlighted in Figure 1, work out a formula for the drag force on a whale in terms of the density ρ of the medium it is moving in, its speed v, and its size ℓ. Specifically, derive the required values of the exponents α, β, γ in the scaling equation

$$F_{\text{drag}} = c \, \rho^\alpha \, v^\beta \ell^\gamma,$$

where c is a dimensionless numerical factor (for the *cognoscenti*, twice the traditional drag coefficient, $c \equiv c_D/2$). You might argue algebraically on purely dimensional grounds, or else give a physical argument for your answer (perhaps having to do with momentum transfer between the animal and the molecules of the surrounding medium).

2. The Cost of Swimming; the Dirigibles of the Sea

A truly fascinating outcome of modern biological inquiry is the discovery of an entire suite of so-called *scaling laws:* empirical relationships that report how key biological and physical observables scale with body size. In fact, scaling laws are far more ubiquitous than in the context of animal body size. For example, at ecological scales, there are a variety of scaling laws that describe how the number of species (and individuals) depend on the size of the island they live on. One particularly important scaling law is shown in Figure 2 and illustrates how metabolic rate (i.e. power) depends upon body mass.

The Power of Resting Humans

Question 2a

Estimate your resting power in watts. Then, look at Figure 2 and comment on how well your estimate for your resting power corresponds to the result predicted by the scaling law.
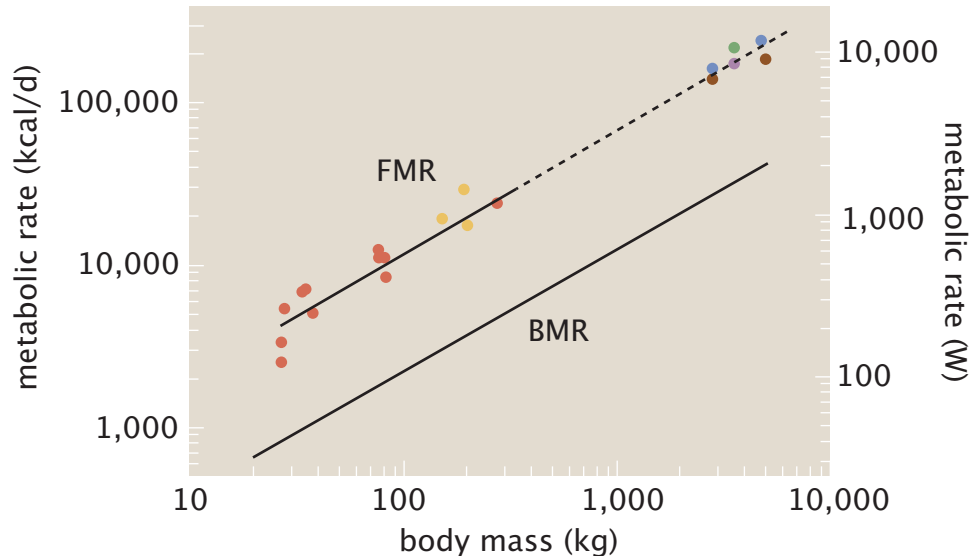
Figure 2: Scaling of metabolic power with animal body size. FMR refers to "Field Metabolic Rate" and BMR refers to "Basal Metabolic Rate". Adapted from Williams *et al.* Ecology, 85(12), 2004, pp. 3373-3384.

Humpbacks and the Cost of Going from Alaska to Maui!

A fascinating aspect of migrations such as that of humpback whales is that they go unimaginably long periods without eating. The great cetaceans, such as the famed humpbacks of Alaska, leave their feeding grounds there and head for a tiny area nestled between Maui, Lanai, and Molokai where they perform another crucial activity. It is here, in the warm waters of Hawaii, that they mate and birth offspring. Recent aerial measurements make it possible to measure the cross-sectional area of the same whale both in its feeding grounds and in its mating grounds, thus allowing us to measure the mass they lose in the process. In this part of the problem, we examine the motion of these whales and their energetic costs and consequences.

For the following exercise, you can assume that hydrodynamic drag scales with the frontal cross-sectional area A and some drag coefficient (some fraction of one). However, as a warning, we note that vexingly, different fields use different conventions for empirically-reported drag coefficients. Specifically, the reference area $A \equiv L^2$ varies by application. For cars, drag coefficients are usually reported assuming that the reference area A is the frontal cross-sectional area; for airfoils/airplanes, the nominal wing area; and for submerged bodies, the total wetted surface area. Since wetted or wing areas are often much

larger than frontal/projected areas, the empirical drag coefficients reported for (air)ships are often much smaller than for cars due to this convention.[1]

Accordingly, if you wish your calculation to be in more direct contact with the biomechanics/hydrodynamics literature, you may use the fact that streamlined whales have been measured to have a drag coefficient of $c_D \approx 3 \times 10^{-3}$ *relative to their total wetted surface area* (see Miller *et al.*, *Journal of Experimental Biology* 207.11 (2004): 1953-1967), which you can approximate as that of a cylinder of comparable characteristic dimensions.

Question 2b

Estimate the fastest speed v_{max} a whale could possibly swim if all of the metabolic power difference between field metabolic rate and basal metabolic rate (of Figure 2) goes into overcoming drag. Assume they swim constantly. First, work out a formula giving that speed in terms of power; then plug in numbers based on your best estimates. For the purposes of substituting numbers into this problem, consider specifically an orca (with length of order 7 m and mass of order 4700 kg).

Question 2c

Compare your estimate for maximal swim speed to actual empirical figures for killer whale swim speeds. These could include two studies by Williams and coworkers: *Journal of Zoology* 256.2 (2002): 255-270 and *Marine Mammal Science* 25.2 (2009): 327-350. Also find a value for the fastest speeds that whales tend to swim. Last, explicitly estimate the factor by which whales increase their metabolic output at this maximum exertion, versus at rest, and compare this to that of humans.

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[1]In fairness, these idiosyncratic conventions for drag coefficients hint at some different dominant physics for these objects—the contribution of a "pressure drag" (imposed by the battering ram of a shape impinging on fluid) scales more nearly with the frontal/projected area of the object, whereas "skin (parisitic) drag" scales more closely with the total wetted area. The total drag of a real object acknowledges both contributions, but one contribution can dominate depending on physical context.
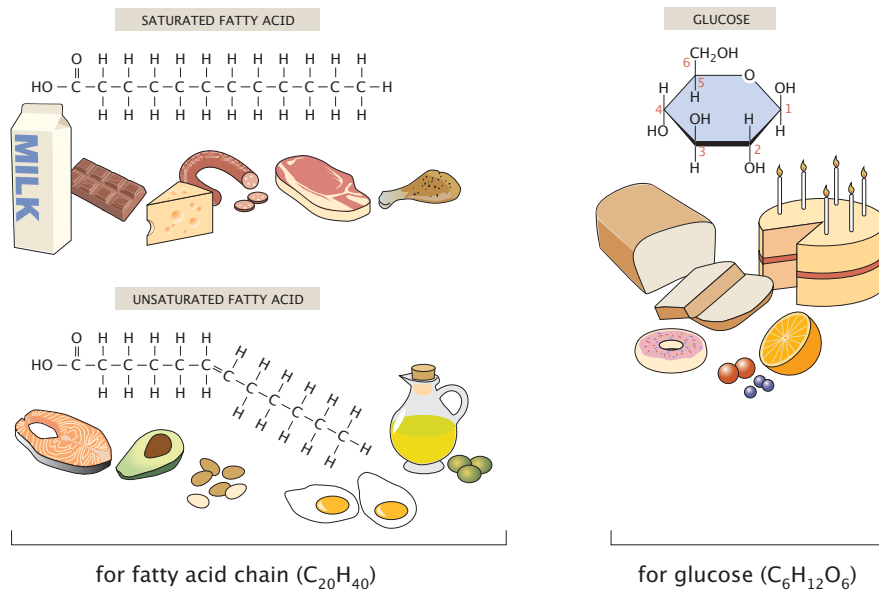
Figure 3: Energy content of fat and sugar resulting in the 9/4 rule.

Question 2d

To estimate how much fat must be burned to give sufficient energy to a migrating whale, first we must know the energy density of fat. This gives us the chance to earn the right to use a useful rule of thumb by deriving it: the energy contents of different foods are about 9 kcal/g (or, 40 kJ/g) for fat and 4 kcal/g for carbohydrates (the "9/4 rule" for fat to sugar, as illustrated in Figure 3).

To find this, we compare the number of carbon-carbon bonds per molecular mass in the two cases. Assign an energy ε to the covalent C-C bonds found in sugars and fat molecules. Now, by counting the number of such bonds per molecule and dividing by the total molecular mass of these molecules, provide an approximate justification for the 9/4 rule.
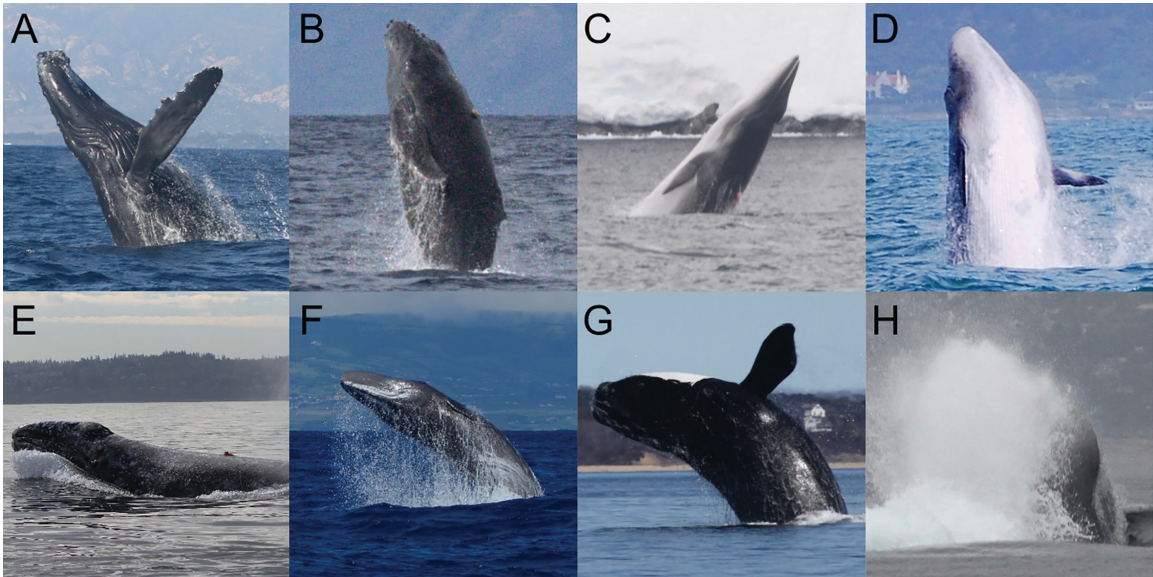
Figure 4: Whales breaching. Adapted from Segre *et al.*, *Elife* 9 (2020): e51760.

Question 2e

By assuming that the energy to overcome the swimming drag force comes from burning fat, work out the mass of fat that has to be burned for a humpback migrating between Alaska and Hawaii and compare that to the mass of the whale at the start of the journey. What fraction of its body weight do you find needs to be consumed?

The Power of a Whale Breach

Remote sensing and tracking gives extraordinary new detail to some of whales' most expressive behaviors. For instance, Segre *et al.* (*Elife* 9 (2020): e51760) measured the depths of humpback whales as they performed incredible breaching ascents and leaped into the air as shown in Figure 4.
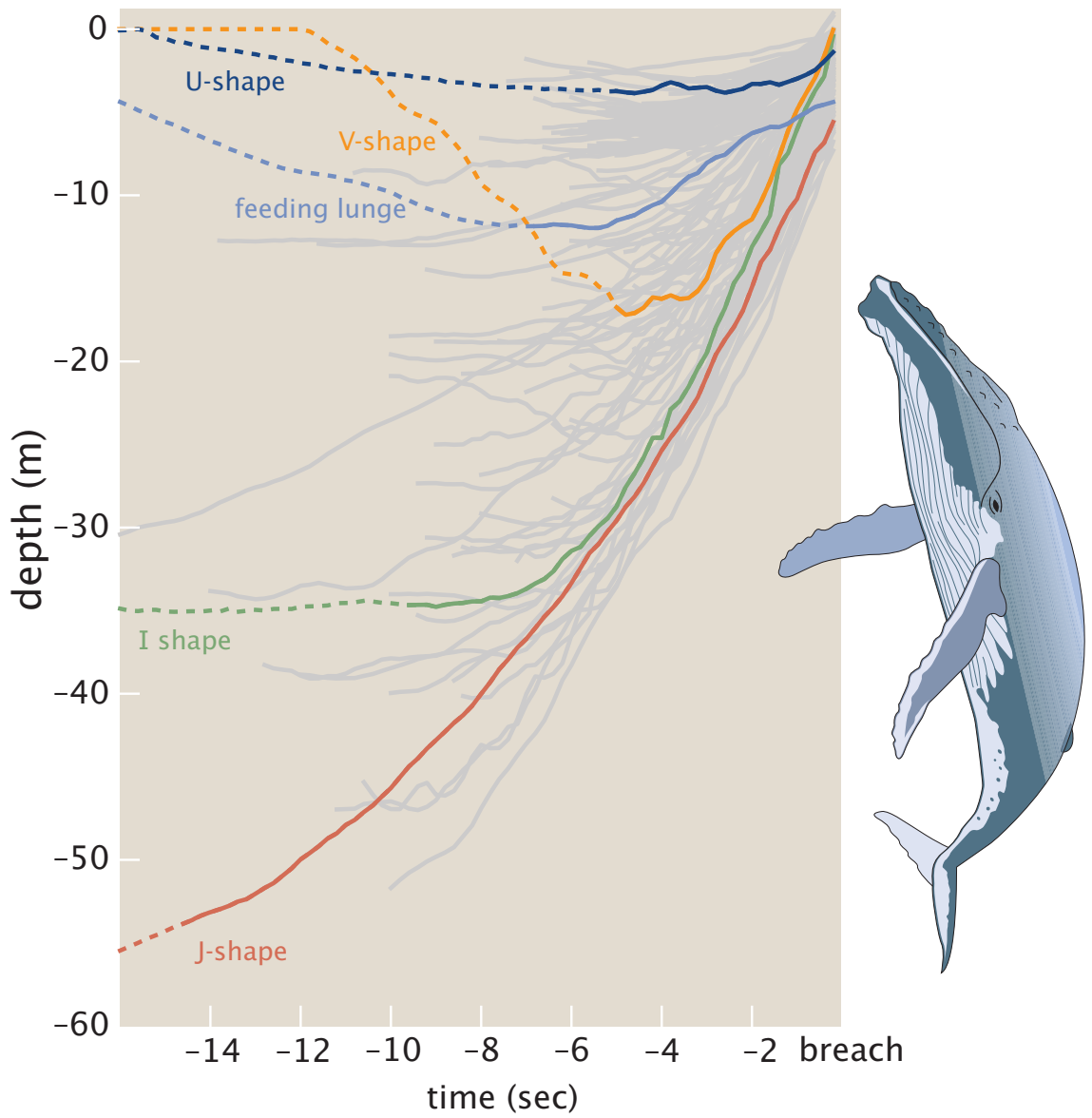
Figure 5: Depth vs time for whales. Adapted from Segre *et al.*, *Elife* 9 (2020): e51760.
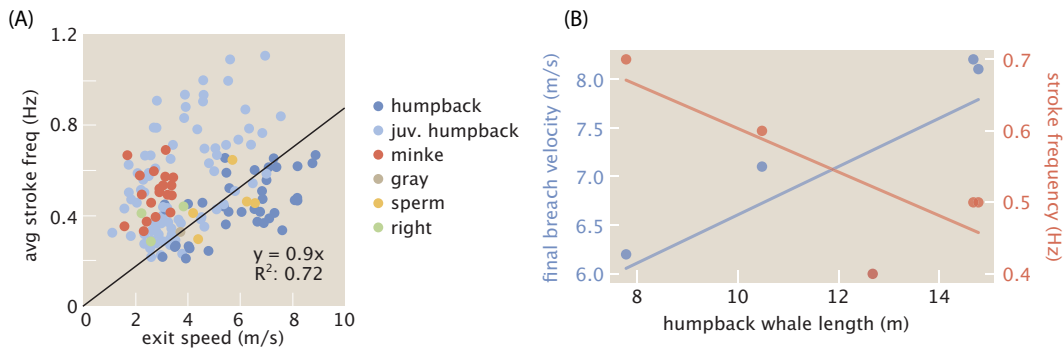
Figure 6: (A) The stroke frequency at which humpback whales swim correlates
with the velocity they achieve as they exit the water. Figure adapted from
Segre *et al.* (*Elife* 9 (2020): e51760). (B) Larger humpback whales tend
to reach faster final breach velocities, even though they stroke with smaller
frequency. (Data were extracted from the same paper's Table 3, which we
linearly regressed separately.)

Thus far, our analysis of whale movement has been completely agnostic to
how they move. We will be able to start correcting this oversight by appealing
to a remarkable regularity of swimming locomotion.

Here are two motivating facts. Segre and colleagues remarked that the
breach speed at which whales exit the water correlates with their average fre-
quency of strokes while swimming (see Fig. 6). They also noticed that longer
whales had a smaller stroke frequency, but tended to reach faster velocities
(see Fig. 6). They do not interrogate these trends further. Armed with a
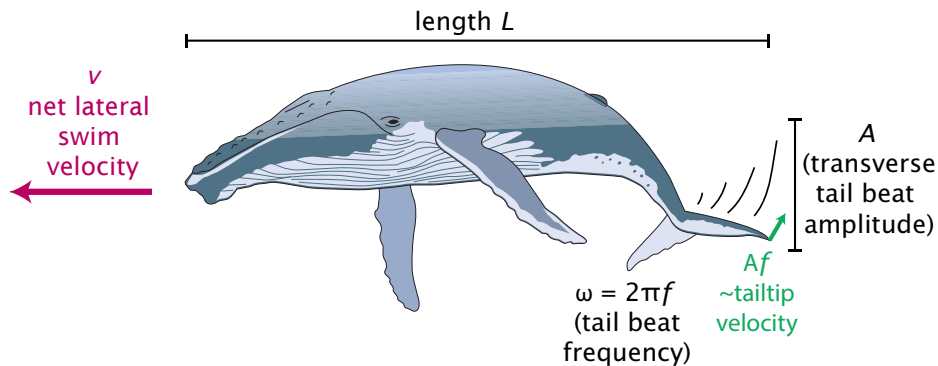scaling argument, we will try to do better and understand why they occur.

Figure 7: Schematic illustrating how the locomotion of a swimmer is related to the net lateral velocity it achieves.

Question 2g

The Reynolds number is a dimensionless number that characterizes important features of fluid flow; it is defined as $\mathrm{Re} \equiv \frac{vL}{\nu}$, where v is the fluid (or swimmer) velocity; L is the swimmer's characteristic length; and $\nu \sim 10^{-6} m^2/s$ is the fluid's kinematic viscosity. In a beautiful paper, Mahadevan and colleagues (*Nature Physics* 10.10 (2014): 758-761) establish that despite the complexity of aquatic locomotion for big fast swimmers, marine swimmers tend to follow a power law $\mathrm{Re} \sim \left(\frac{\omega A L}{\nu}\right)^{1}$ for large Reynolds numbers (turbulent flows; big fast swimmers), but $\mathrm{Re} \sim \left(\frac{\omega A L}{\nu}\right)^{4/3}$ for low Reynolds numbers (laminar flows; small slow swimmers), where ω is the tail beat frequency, and A is the tail beat amplitude (See Fig. 7).

Infer how $A\omega$ scales with size L and velocity v for a whale, versus a small fish. (This quantity is essentially the speed at which the tail sweeps transversely; it sets the momentum of a whale's tail if it smacks you.)

This is a remarkable relation, because it says that if the only thing you know is the speed at which a marine creature is swimming, you also know *how* it is swimming, e.g. the extent and transverse speed of its contortions as it wiggles through fluid.

What does this scaling relation anticipate about the ways that whale swim? The Segre data do not specifically report amplitude A; this would likely be hard to measure from afar, though you yourself will be able to estimate it (in units of body lengths) by casually watching a whale swim. But assume that the amplitude A of a whale's tail sweep is proportional to the total body length L. Then this scaling predicts that the true natural variable connecting a whale's stroke frequency and lateral swim velocity is $A\omega \propto L\omega$, which should be just proportional to the velocity a whale exhibits. Peering into the Segre data, we find this is indeed the case!
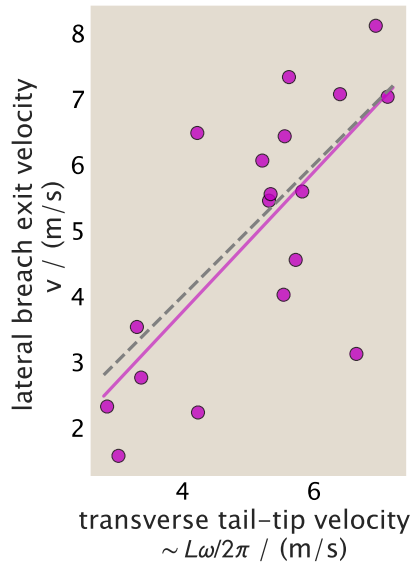

Figure 8: Analyzing a subset of Segre's whale trajectories (for which length, velocity, and stroke durations are simultaneously defined) affirms the scaling law that Mahadevan and colleagues predict. (The best-fit line (slope ≈ 1.06) is in solid purple; it is close to the $y = x$ (slope 1) line (dotted grey) shown for reference.)

Returning to the phenomenologies that motivated us, the first scaling between average stroke frequency and exit speed that Segre *et al.* reported in Fig. 6 can make sense from the primordial scaling we expect from Mahadevan and coworker's reasoning. If, over these wide data, the length L of whales varies more slowly than the stroke frequency ω across individuals, then the $v \sim L\omega$ relationship confirmed in Fig. 8 reproduces the general $v \sim \omega$ correlation reported. Further, the spread about that correlation probably has something to do with the fact that length also varies and matters across whale

individuals.

We extracted two other phenomenologies in Fig. 6, grounded in a narrower subset of Segre's whale observations: final velocities increase by a factor of about 30% when whale length about doubles; and whale swim stroke frequency decreases by a factor of about 75% as whale length doubles. These relationships might be explained by conjecturing that longer whales actively choose slower stroke frequencies ω so as to *more* nearly maintain the final breach frequencies v they achieve, but do not modulate their stroke frequencies quite dramatically enough to cancel out their larger modulation of length, yielding that still-discernible (albeit reduced) 30% modulation in breach velocity.

> **Question 2h (extra credit)**
>
> By imagining that submarines are not so different from whales from the perspective of drag, argue how much gasoline fuel a submarine would need to go around the globe (while submerged), assuming it travels a whale-like speed of a few meters per second. Thus infer the prominence of *nuclear* submarines.

Moving on the Serengeti

As discussed in class, one of the most curious outcomes of a careful quantitative analysis of the population of wildebeest on the Serengeti was the discovery that over a roughly decade long period, the number of wildebeest more than quadrupled from several hundred thousand to more than a million. In the 1950s and 1960s, the observations that were made to count the wildebeest population were carried out by small propeller planes flying transects over the plains of the Serengeti. Amazingly, in the modern world, these laborious studies have been complemented by the use of satellites and drones. In this problem, we will make a rudimentary analysis of the use of satellite imaging to identify animals such as wildebeest and elephants. As we will see, our analysis will be insufficient to handle animal detection in diverse settings; later in the term, we will generalize the approach used here to do a similarly rudimentary analysis with machine learning.

Figure 9: Satellite image of elephants near a watering hole. We are using this as a simple first example just to practice handling images in Python, and considering the most rudimentary approaches to identifying objects in such an image.

> ## Question 3a
>
> Along with this homework, we've provided two images of elephants meandering around two very distinct environments. These satellite images, with an incredible resolution of 31 cm x 31 cm, were ultimately used to train a convolutional neural network capable of identifying elephants across an array of heterogenous environments. But why was machine learning necessary in the first place? Using the first Python tutorial for this homework as a guide, develop a simple algorithm to detect elephants in the image of the elephants near the watering hole. Explain your process, assumptions, thresholds, and the ultimate accuracy of your approach.
>
> While preparing your solutions, please use the template Colab notebook provided on the course website.
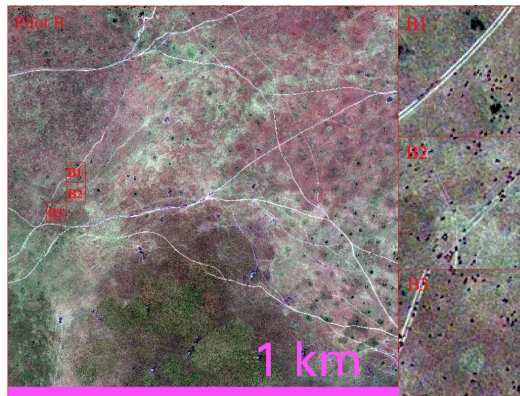
Beasts, Wild and Tame

Figure 10: Yang and colleagues analyzed high-resolution (GeoEye-1) satellite images of plains captured from space; this is a slightly-animal-dense example. While they used deep learning to detect wildebeest (black dots in the inset panels B1-B3 at right), we can consider the image manually by eye here and estimate a population. (Note that zooming in helps suggest that the inset panels indicated probably contain nearly all of the wildebeest in the whole 1 km×1 km main image square.) Adapted from Fig. 2 in Yang *et al.*

Recall that in our first lecture, we saw data reporting that the modern population of wildebeest in the Serengeti was about $N_{\text{wildebeest}} \approx 1.5 \times 10^6 \equiv$ few $\times 10^6$ wildebeest in the 2010-2020s (see also a discussion by Mduma *et al.*, *Journal of Animal Ecology* 68.6 (1999): 1101-1122.). Estimate the daily grass eaten by the migrating wildebeest population. Make a corresponding estimate of the area of grassland grazed on each day. Last, estimate the time scale for the wildebeest population to consume all the grass in the Serengeti if it did not grow back.
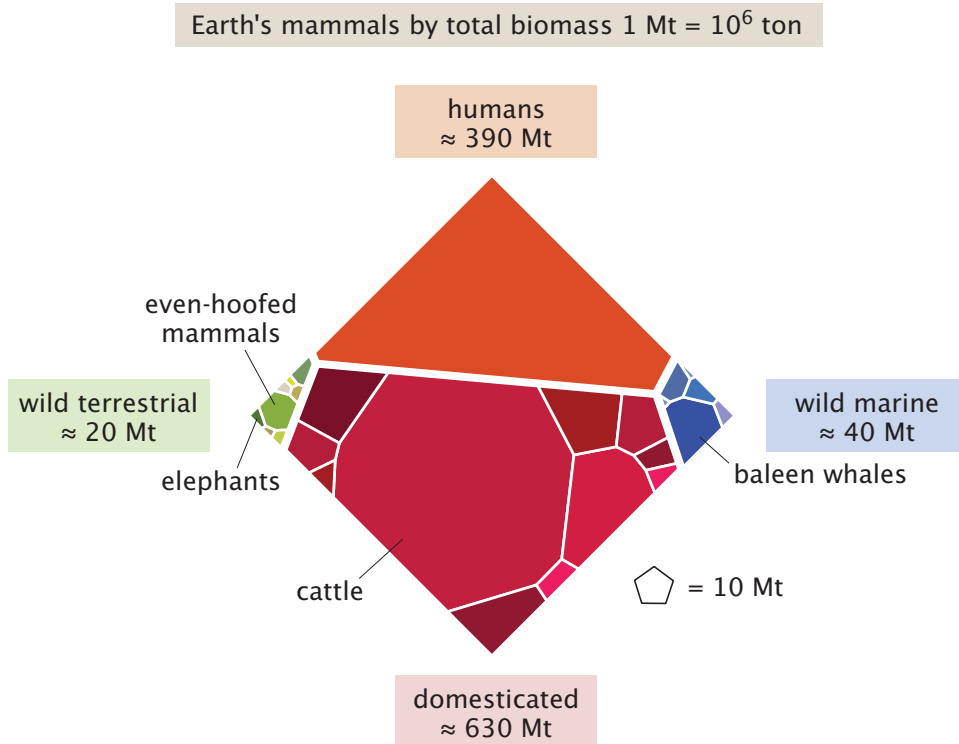

Figure 11: The biomass of mammals. Humans and their livestock dominate the mammalian animal biomass.

As seen in Figure 11, one might say we live on a planet of cows. Given that the argument was made that the wildebeest carry out their migration precisely for the advantage of maximizing their food uptake, it is of great interest to contrast the foraging habits of wild mammals and their domesticated counterparts.

Question 4c

Estimate the daily grass eaten by grazing cattle. How much land does this correspond to for each cow? Make an estimate of the total number of beef cows on Earth and how much land it takes to feed them each year.