Superpowers Beyond the Reach of Kryptonite

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"I only went out for a walk and finally concluded to stay out till sundown, for going out, I found, was really going in." - John Muir

Abstract

One of the signature achievements of centuries of effort in the natural sciences and mathematics is the establishment of "fundamentals" that have served as the foundation for all science thereafter. In this essay, I will argue for a semantics of "what is fundamental" defined not on the basis of a reductionist search for explanations based upon microscopic constituents, but rather on those deeply satisfying insights that are known for their broad explanatory reach. This explanatory reach can be thought of as an intellectual superpower because possessing it allows us by pure thought alone not only to explain things that are already known, but to predict things that are not yet known. Unfortunately, fundamentals have a darker side as well. The addition of the three simple letters "-ism" takes the notion of fundamental and turns it into one of the worst of human traits, namely, the insistence that there is only one divinely-inspired truth. As an antidote to such fundamentalism, I close by reflecting on one of the most fundamental lessons of science: the requirement for the kind of simultaneous openness and skepticism that makes science work.

What Constitutes Fundamentality?

Richard Feynman opens the famous lectures that bear his name with the observation: "If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words? I believe it is the atomic hypothesis that all things are made of atoms." The atomic theory is clearly among the most fundamental of things we can say about the world. But others have laid a claim for the fundamental. A search on Amazon for books with the title "Fundamentals of X" yields more than one-hundred pages of X's including X = Tattoo, Marijuana Horticulture, Tarot, Nursing (perhaps the most common one), The Faith, Physics (over and over), Pathology, Chess, Interior Design, English Riding, Soviet Weightlifting, Deep Learning, Fly Casting, Japanese Swords, Defensive Revolver, Hawaiian Mysticism, and even, Mobile Heavy Equipment.

What we see from this truncated list is that one of the ways of viewing fundamentals is as the stuff that anyone pretending competence in a given subject area needs to know. But the semantics of "fundamental" are complicated with many accepted uses of this word, only one of which is exemplified by the Amazon search. Clearly not all competencies are created equal and the deep atomic competency referred to by Feynman has much broader reach than that of competency in Soviet weightlifting. Another view of the concept of fundamental hinted at in the Feynman quote focuses on explanations in terms of the constituents of matter. The study of these fundamental particles and their interactions forms one of the major canvases upon which 20th century physics was painted. Biology has a similar notion of fundamental based on the huge successes of molecular biology, in which it is argued that the way to understand biological mechanism is by figuring out what the molecules are doing.

But there are other ways to think about the fundamental without making reference to the microscopic constituents of a system. Here I will argue for a definition of fundamental that subsumes the idea of elementary constituents, but goes much further. Specifically, the criteria I will invoke for deciding what is fundamental are whether or not a given result has explanatory reach, whether that result gives that ineffable (and subjective!) feeling of being deeply satisfying and whether that result is actionable. As we will see later in the essay, precepts such as the conservation of mass, momentum and energy have enormous explanatory reach and serve as examples of the notion of fundamental showcased here. Classic descriptions of chemical kinetics, elasticity and hydrodynamics all appeal to these fundamentals as the bedrock upon which the subjects are built. But more importantly, when confronted with new kinds of matter - whether liquid crystals or granular media or the mitotic spindle - these very same conservation laws are actionable in the sense that they form the foundation for further quantitative thinking.

Let's examine the profound explanatory reach of several historic fundamental insights.

Taking Up the Mantle of the System of the World

Mathematics is full of fundamental results as I have defined that idea above. *The Elements* of Euclid is packed with them. A beautiful example is Proposition 20, from Book 1 which I show in Figure 1. This Proposition says: "In any triangle two sides taken together in any manner are greater than the remaining one"[1]. The reader is urged to absorb the proof in Euclid's original. But what is it that makes this result so fundamental? Emanating from this one, apparently trivial result, which many might think doesn't even require proof, are intellectual tentacles that permeate all corners of mathematics and the natural sciences. Let me focus on just one of those tentacles here, the connection to all the different ways scientific principles are formulated as minimization or maximization problems that inspired Euler to say "Nothing takes place in the world whose meaning is not that of some maximum or minimum" [2].

There is a direct intellectual path from Proposition 20 of Euclid to the question of the shortest path between two points as formulated by Heron (see below), to Fermat and the principle of least time in optics, to the variational principles of mechanics, and from there to the path integrals Feynman developed to think about the quantum world [4]. The famed Heron problem (see Figure 1(B)) poses the challenge: "A and B are two given points on the same side of a line l. Find a point D on l such that the sum of the distances from A to D and from D to B is a minimum" [3]. Beautifully, brilliantly and simply, the solution to this problem exploits Euclid's Proposition 20 by noting that the shortest path can be just as well explored by finding the distance to the point B_1, which is the reflection of the point B around the line l. Now we see Euclid's proposition in action, since any position of D other than the one that leads to a straight line from A to B_1 results in two sides of a triangle that are manifestly, by Euclid's Proposition 20, longer than the edge AB_1.

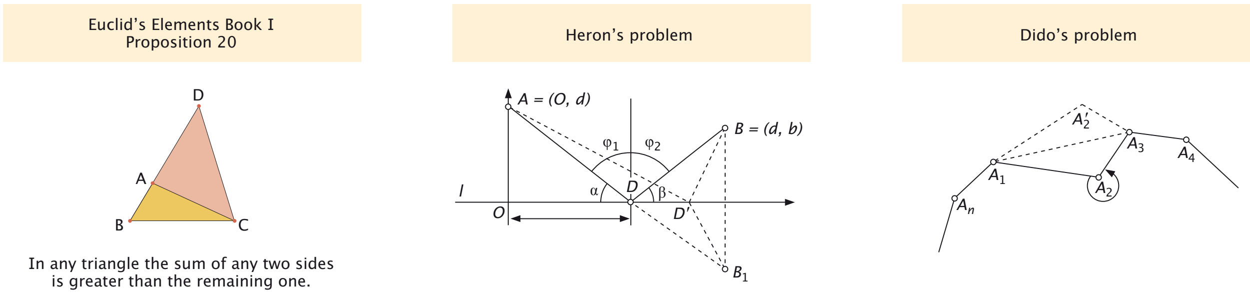

Figure 1. Euclid and the broad reach of the triangle inequality. (A) Proposition 20 of Book 1 of Euclid's "Elements" provides a beautiful, strictly geometric proof of the triangle inequality. (B) Heron's problem [3]. See text for explanation. (C) Dido's problem. Here we show the discrete approximation to the closed plane curve by a series of line segments. Parts (B) and (C) adapted from ref. [3].

With the solution to Heron's question in hand, we can now solve other interesting problems including the first optimization problem known as Dido's problem [5]. In modern language, this can be stated as "Among all closed plane curves of a given length, find the one that encloses the largest area [3]," though the original story of the resourceful princess Dido is much more interesting. There, as Virgil tells us in *The Aeneid*, the Phoenician princess Dido was fleeing her murderous brother, Pygmalion, and found herself in what is now Tunis, where she was granted a plot of land that could be encircled by a bull's hide. Cleverly, she proceeded to cut the hide into a thin strip and solved the resulting isoperimetric problem [3]. The solution hinted at in Figure 1(C) is the treatment of Zenodorus that tackles the corresponding problem, "If there exists a plane n-gon having largest area among all n-gons of given perimeter, then it must have equal sides and equal angles." Disguised in the answers to these problems are other treasures such as the laws of optics (e.g. the laws of reflection and refraction), Newton's proof of Kepler's equal area law and the philosophical underpinnings that gave rise to the importance of minimum principles.

To get an idea of how the kind of explanatory reach shown in the geometry of Euclid works in the analysis of the physical world, we turn to the paradigmatic insights of Newton linking both terrestrial and celestial mechanics. Indeed, Figure 2 shows one of the most far-reaching scientific figures that I have ever seen. In this one image, Newton tells us how the mechanics of projectiles conceived by Galileo and the mechanics of planetary orbits developed by Kepler are really simple consequences of his second law of motion, $\mathbf{F} = m\mathbf{a}$, and the law of universal gravitation. Through laborious and ingenious experiments, Galileo had worked out the kinematics of projectile motion culminating in the realization that the trajectories are parabolas. At about the same time, Kepler was engaged in figuring out the kinematics of planetary motion through careful analysis of the orbit of Mars. His efforts culminated in the assertion that planets travel in ellipses with the sun at one of the foci and that equal areas are swept out by their orbits in equal times. Newton's fundamental insights made it possible to bring together under the same intellectual roof these phenomena that, at first glance, seem completely distinct [6].

In the third book of his monumental *Principia*, boldly entitled "The System of the World", Newton gives us Figure 2, showing us what the definition of fundamental as I am using it

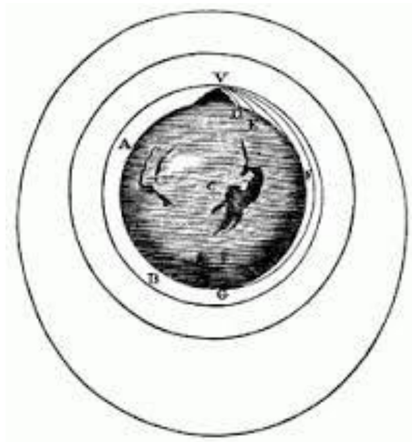

Figure 2. Figure from Newton's "System of the World" where he says of the projectile that it can "be made to describe a curved line in the air; and through that crooked way is at last brought down to the ground; and the greater the velocity is with which it is projected, the farther it goes before it falls to the earth...till at last, exceeding the limits of the earth, it should pass into space without touching it." And thus are Galileo and Kepler's great ideas joined in intellectual matrimony by Newton's understanding of gravity.

actually looks like. Book III provides a template for what was to come in the physics thereafter. Few have taken up the mantle of Newton's System of the World more successfully than James Clerk Maxwell. In the chapter of his *A Treatise on Electricity and Magnetism* entitled "The Electromagnetic Theory of Light", Maxwell shows us how the seemingly distinct phenomena of electricity, magnetism and light are all manifestations of the same physics. There, Maxwell articulates his views on the electromagnetic origins of light where he notes "If it should be found that the velocity of propagation of electromagnetic disturbances is the same as the velocity of light, and this not only in air, but in other transparent media, we shall have strong reasons for believing that light is an electromagnetic phenomenon." It is very hard from our current vantage point to see how bold and far-reaching this insight really was. The problem with our human experience of the world is no matter how hard we try, we fall into the trap of taking things for granted. A flight across the Atlantic Ocean from Europe to North America with views down on the bergschrunds of Greenland is so routine that nearly every passenger on a modern jetliner has their window shades down so they can sleep or watch a movie. The intellectual equivalent is how easily now in physics we take for granted Maxwell's stunning unification of the phenomena of light with the laws of electricity and magnetism.

As part of his musings about electricity and magnetism, Maxwell had arrived at an equation for the spatiotemporal dynamics of the electromagnetic field and this equation is of the now familiar form that describes the vibration of a guitar string, namely,

$$\frac{\partial^2 y(x,t)}{\partial t^2} - \frac{T}{\rho}\frac{\partial^2 y(x,t)}{\partial x^2} = 0, \tag{1}$$

where $y(x,t)$ is the displacement of the string at position x at time t, T is the tension in the string and ρ is its density. By dimensional analysis, we learn in our first encounter with such

787.] In the following table, the principal results of direct observation of the velocity of light, either through the air or through the planetary spaces, are compared with the principal results of the comparison of the electric units :—

Velocity of Light (mètres per second).		Ratio of Electric Units (mètres per second).	
Fizeau	314000000	Weber	310740000
Aberration, &c., and Sun's Parallax	308000000	Maxwell	288000000
Foucault	298360000	Thomson	282000000

Figure 3. Maxwell considers the fundamental quantities of electricity and magnetism and what they imply about the speed of light. He compares the measurement of the speed of light (left column) and the quantity constructed from electromagnetic units that has dimensions of velocity (right column).

equations that they feature a velocity, in this case $V = \sqrt{T/\rho}$. Maxwell tells us his insights into this question noting: "The quantity V ... which expresses the velocity of propagation of electromagnetic disturbances in a non-conducting medium is ... equal to $\frac{1}{\sqrt{K\mu}}$." In modern parlance, this is what we would write in SI units, as $\frac{1}{\sqrt{\epsilon_0\mu_0}}$. Of course, such an amazing insight led Maxwell to wonder how well the velocity that emerges from electromagnetic units of the dielectric constant ϵ_0 and the magnetic permeability μ_0 will agree with the measured speed of light. As a result of the analysis presented in Figure 3, Maxwell modestly concludes, "our theory, which asserts that the two quantities are equal, and assigns a physical reason for this equality, is certainly not contradicted by the comparison of these results such as they are."

Fundamentals as I have defined them here are certainly not restricted to mathematics and physics. While wandering around the jungles of the Malay Archipelago, Alfred Russel Wallace realized that his experiences both in the Amazon and in the islands of Southeast Asia pointed to a fundamental insight about the living world, codified in his so-called Sarawak Law which states: "Every species has come into existence coincident both in space and time with a pre-existing closely allied species." That is fundamental. Why? Because it links all of life, both living and extinct, in one vast narrative. With the short paper that introduced his Sarawak law, Wallace had discovered the *fact* of evolution. Shortly thereafter, in a communique written in a feverish malarial stupor from the island of Ternate, Wallace would follow with his version of the *theory* of evolution [7].

Just like there is a direct intellectual path from Proposition 20 of Book 1 of Euclid's *Elements* to our modern thinking on quantum mechanics, there is a similar line linking Wallace's fundamental biogeographical insights to our modern understanding of life on our planet as we see it today. For example, one of the fascinating puzzles of modern biogeography is the unexpected presence of amphibians on oceanic islands. In one of these cases, that puzzle is resolved through the realization that these amphibians have traveled on "freshwater paths" in the ocean due to huge quantities of freshwater deposited in the Gulf of Guinea by the Congo River [8]. A modern day example of such long-distance dispersal is offered by the biological debris that has been peppering the west coast of the United States in the time since the 2011 Fukushima earthquake [9]. Hundreds of new species have arrived on the shores of Oregon and Washington, precisely as Darwin had hypothesized long-distance dispersal would lead to adaptive radiations.

One of the most powerful windows onto the all embracing fundamentals of the theory of evolution is the genome. The puzzle of how land mammals could have made the transition to a strictly aquatic lifestyle as whales was one of the mysteries of evolution highlighted by Darwin in *On the Origin of Species*. But, if our understanding of evolution is correct, then we should be able to use our evolutionary superpower to make predictions about how the genomes of these animals have evolved. One such prediction is that whales, and other placental mammals that either lack teeth altogether or have teeth without enamel, should harbor molecular fossils in the now dysfunctional genes responsible for the synthesis of enamel [10]. In the same way that vestigial limbs were predicted (and found) in fossil whales, we expect vestigial DNA in the context of defunct molecules such as enamelin, one of the key enzymes in enamel synthesis. Chapter 32 of Melville's *Moby Dick* is justly entitled "Cetology" and exhaustively, but poetically, tells us about the many whales that people the oceans of the world. Of the right whale, he tells us "In one respect this is the most venerable of the Leviathans, being the one first regularly hunted by man". The DNA sequence of this Leviathan has since revealed, as predicted by our modern understanding of molecular evolution, a defunct gene for the protein enamelin. Indeed, if we compare the functional *enamelin* gene of the pig which between positions 1239-1247 reads TGTTTACTA, to that of Melville's venerable right whale, we find a so-called frameshift mutation, TGTT-ACTA, that results in a garbled and nonfunctional protein. Further comparison of the same gene in these two very different animals reveals that between positions 2501-2512, whereas the pig has the letters TGG, in the whale they have become TGA, the ominous "STOP" signal that prevents a functional protein from being made. Evolutionary thinking is a superpower that allows us to anticipate the character of DNA sequences, even before those sequences are known [11].

Kill the Atoms

In his classic piece "More is Different" (what discussion of fundamentals in science could neglect to mention it!), Philip Anderson demonstrates the flaws in thinking that subject X is "just" applied subject Y [12]. The theory of elasticity gives us a chance to see Anderson's assertion in action. One way of thinking about mechanical equilibrium is in the language of forces, but an equally potent approach is the idea that equilibrium reflects the minimum of some potential energy. The fundamental idea of elasticity is that if we disturb the system, forcing it to make some small excursion away from that equilibrium point, the resulting energy will be a quadratic function of the geometric degrees of freedom that characterize that excursion. Those geometric degrees of freedom are no longer the individual atomic positions, but rather the components of the strain tensor ϵ_{ij}, signaling that we are operating at a new coarse-grained level. In its most general form for a linear elastic solid, we say that the energy stored per unit volume of material is given by $W(\epsilon) = \frac{1}{2}C_{ijkl}\epsilon_{ij}\epsilon_{kl}$, where C_{ijkl} is the elastic modulus tensor.

This same kind of coarse-graining had already served as a fundamental outcome of Newton's thinking on gravitation [6]. One of the deep questions that puzzled Newton was how to think about the gravitational interaction between an extended object such as the earth and the famed apple that falls from the tree to the ground below. The problem is that every little material element of the earth should itself attract that apple according to the inverse-square law. Since those parts of the earth closer to the apple will attract it more strongly than those points on the earth that are farther away, it was not at all clear how that would jibe with the idea of treating the earth as a point object with all of its mass concentrated at the center, as Newton had done in his earliest approximate investigations of the problem. In his so-called Superb Theorems, Newton rigorously demonstrated how what appeared to be a simplifying assumption is in fact a fundamental truth, namely, that a spherically symmetric body will act gravitationally as though all of its mass is concentrated at the center. Amazing and fundamental.

Coarse-grained thinking shows its power again in the broad class of dynamical equations that describe how probability distributions evolve over time. One of the most celebrated examples is the Boltzmann kinetic equation, which applies to processes ranging from the collision of galaxies to the change of density of plasmas to the motion of electrons in solids. Chemical master equations similarly focus on the time evolution of some phenomenological set of degrees of freedom giving us dynamical equations for the probability distributions of these degrees of freedom [13]. These equations are relevant in contexts ranging from the frequency distributions of different alleles in evolutionary biology to the dynamics of cytoskeletal assembly. In each of these cases, the underlying microscopic degrees of freedom that are presumed to be the real basis of the observed phenomena are superseded by more macroscopic degrees of freedom, though the dynamics at smaller scales informs the phenomenological higher-level description through effective parameters such as the elastic moduli C_{ijkl} introduced above. All of these examples make it clear that huge swaths of the phenomena of nature can be described by fundamentals that make no reference to the underlying microscopic degrees of freedom and that indeed, more is different.

The Weight of Inevitability: Hidden but Required

Many of the most fundamental insights into the world around us are those things that are true because they must be true. The best way to see this might be by example. Physics often uses continuous symmetries to establish deeply fundamental results. The theorem of Emmy Noether famously tells us how to deduce conserved quantities such as energy and momentum from symmetries. The requirement that the very same equations of Maxwell I discussed earlier must be invariant under certain classes of transformations leads directly to the ideas of relativity theory. Thus symmetry reveals hidden but inevitable truths.

Discrete symmetries provide a fascinating and important case study too. Take, for example, transformations of an equilateral triangle that leave it unchanged. We are all used to the idea that we can rotate an equilateral triangle by $2\pi/3$ and it will look exactly as it did before. Similarly, we could have rotated it by $4\pi/3$ with the same outcome. Amazingly, a simple knowledge of the so-called group of symmetry transformations that a given object can undergo while leaving the system unchanged - whether it's a molecule of benzene or buckminsterfullerene - will already tell us much about the spectrum of electronic or vibrational states that molecule will have [14]. Whether we are trying to learn the vibrational frequencies of one of these molecules on the basis of its stiffness matrix or the spectrum of energy eigenvalues that follow from the Schrödinger equation, these problems can be thought of as finding the eigenvalues of a matrix. The group theory superpower in cases like this allows us to know ahead of time, without ever diagonalizing one of these matrices, how many of the eigenvalues of that matrix will be the same, and even to know much about the character of the corresponding eigenvectors. Similarly, the elastic modulus tensor C_{ijkl} we considered in the previous section looks at first blush as though it has $3 \times 3 \times 3 \times 3 = 81$ components. But symmetry collapses that apparent proliferation of parameters by telling us that for isotropic materials there are only two such elastic constants and for cubic materials, only three. Symmetry again tells us of hidden insights that must be true.

There is a similar kind of inevitability to the probability distributions we use to describe many of the apparently random features of the world, whether the height of human beings or the distribution of mutations across their genomes. A particularly appealing perspective is to think of each of these different distributions in terms of a fundamental story [15]. For example, in the combinatorics central to the binomial distribution, one can give a “story proof” telling us that the number of ways of choosing 4 members of a team from ten people is the same as the number of ways of choosing 6 members of the non team from that same ten people, thus proving that $\binom{10}{4} = \binom{10}{6}$. The rise of the Gaussian distribution (i.e. the bell curve) is perhaps the most celebrated probabilistic example of a kind of fundamental inevitability. Specifically, if we pick a bunch of random numbers and take their average, we get a new random number. If we now repeat this lots of times, the collection of random averages we generate will have a Gaussian distribution. Though this result might seem abstract and far removed from the concerns of real world science, in fact, it informs us about random-walk problems in contexts as distinct as the theory of the meanderings of a polymer of DNA to how to think about diffusion of carbon atoms in steel.

One of the central missions of science is the deliberate search for these kinds of inevitable truths, many of which provide the fundamental framework serving as a trellis for different disciplines in science.

Fundamentals and Fundamentalism

As the examples throughout this essay have shown, there are at least two ways fundamental is commonly used in science, namely, fundamental constituents and fundamental principles. The bridge between these two visions of fundamentality is the prejudice that somehow knowing the fundamental constituents might help us better know the fundamental precepts. I have argued thus far for an expansive definition of the fundamental that goes beyond explanations in terms of microscopic constituents. In this final section I will now turn to a vision of fundamentality that is more a state of mind than any particular kind of intellectual insight or result. At the very root of mathematics and the sciences lies the *process* by which we discover and substantiate the correctness of our thinking about how the world works. The scientific approach to understanding the world provides us with formal rules for deciding whether something merits elevation to the status of knowledge. What is fundamental to science is the rule of reason, the nod to experiment as the court of final appeal and the humility to willingly amend our understanding through successive approximations.

But the very human search for fundamentals has a dark side. How can three letters, -ism, so pervert one of mankind’s greatest activities - the search for meaning in this world - and replace it with one of humanity’s most despicable traits, namely, fundamentalism? Fundamentals embrace the human uncertainty about the world while celebrating our victories in tackling the great unknown. Fundamentalism feeds on precisely the opposite, a presumed and fallacious certitude. Our knowledge of the world reminds me of the unfinished sculptures from Michelangelo that adorn the entry hallway to the Galleria dell’Accademia in Florence (see Figure 4) at the end

Figure 4. Michelangelo's hall of prisoners. Michelangelo's famed David is seen at the end of the hallway, while his unfinished statues line the passage. Science lives with the ambiguity of an unfinished but deeply beautiful understanding of the world around us, favoring that over the illusion of perfect certainty claimed by fundamentalism.

of which stands the majestic, dominant and perfect David. Each new fundamental insight in mathematics and the sciences is like exposing another limb of the emerging forms busting forth from the granite of those beautiful unfinished statues. Fundamentalism pretends at a perfect and complete knowledge, but instead of the perfection of the David, reveals an intellectual deformity.

Fundamentals help us find coherence in the chaos of the world around us, or to figure out at least, that it is chaos! Their possession puts superpowers beyond the reach of kryptonite into the hands of anyone willing to master them. Here, I have tried to remember that the world all around us remains full of some of the most beautiful phenomena and deepest mysteries in the known universe, waiting to teach us many more fundamental lessons to come. That which is fundamental makes our spirits soar, brings a joyfulness to our observation of the world while delivering us from mysticism and magical thinking. The quote from John Muir that opens this brief essay is a reminder of the fundamentals that are all around us if we only but notice them.

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